Optimal Decentralized Management of a Renewable Resource

9th LAGV

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Overview

We consider a fishery exploited by several countries, with our main example based on Levhari and Mirman (1980).

We first analyse the unregulated open-access and cooperative situations.

We then define a new economic mechanism, which induces, with the dynamic of the fish population, a difference game.

We show that (SMNE = Stationary Markovian Nash Equilibrium) :

1. Using the example of Levhari and Mirman, the difference game admits a SMNE;
2. Under a general setting, any SMNE realizes an optimal consumption path of the resource.

Our notations will be:

$$\begin{aligned} x &= \text{Current stock of fish,} \\ F(x) &= \text{Next period's stock} \\ &\quad \text{as a function of the current stock,} \\ \\ n &= \text{Number of countries,} \\ \delta &= \text{Common discount factor,} \\ \\ c_i &= \text{Country } i\text{'s present consumption,} \\ u_i(c_i, x) &= \text{Country } i\text{'s utility.} \end{aligned}$$

Under the unregulated open-access situation, we suppose that:

Each country decides its harvest time-path,
in order to maximize the sum of its discounted utility,
taking into consideration:

- the long-run effect of its present catch
- and the catch of the others countries.

This situation can be formalized as a difference game (Clemhout and Wan, 1979; Levhari and Mirman, 1980).

A (stationary Markovian) strategy of i is a function:

$$s_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+,$$

which associates states x of the fish stock
with consumptions $c_i = s_i(x)$ of country i.

A strategic profile is a vector $\mathbf{s} = (s_i)_{i=1}^n$.

It is said to be feasible if it satisfies, for all x:

- $s_i(x) \geq 0$, for all i,
- $\sum_{i=1}^n s_i(x) \leq x$.

Given an initial state $\varkappa$ and a (feasible) strategic profile $\mathbf{s}$, the sum of player i's discounted utility is:

$$w_i(\mathbf{s}, \varkappa) = \sum_{t=0}^{\infty} \delta^t u_i(c_i(t), x(t)),$$

$$\begin{aligned} \text{where:} \quad & x(0) = \varkappa, \\ & x(t+1) = F\left(x(t) - \sum_{i=1}^{n} c_i(t)\right), \quad t = 0, 1, 2, ..., \\ & c_i(t) = s_i(x(t)), \quad i = 1, ..., n, \quad t = 0, 1, 2, ... \end{aligned}$$

A strategic profile $\mathbf{s}^*$ is a SMNE if, for all $\varkappa$:

$$w_i\left(\mathbf{s}^*, \varkappa\right) \geq w_i\left(\left(\mathbf{s}^* / s_i\right), \varkappa\right), \text{ for all } i \text{ and } s_i,$$

where $\left(\mathbf{s}^* / s_i\right) = \left(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_n^*\right)$.

We admit now that the countries commit to a cooperative management of the fishery.

Formally, we suppose that:

A central planner determines the countries' catch rates,
in order to maximize the discounted sum of all countries' utilities,
taking into consideration the long-run effect of the overall present catch.

A policy $\boldsymbol{\pi} = (\pi_i)_{i=1}^n$ is a sequence of n functions:

$$\pi_i : \mathbb{R}^+ \to \mathbb{R}^+,$$

each of which associates states x of the fish stock with consumption $c_i = \pi_i(x)$ of country i .

It is said to be feasible if, for all x:

- $\pi_i(x) \geq 0$, for all i,
- $\sum_{i=1}^n \pi_i(x) \leq x$.

Given an initial state $\varkappa$ and a feasible policy $\boldsymbol{\pi}$,
the sum of all players' discounted utility is:

$$W(\boldsymbol{\pi}, \varkappa) = \sum_{t=0}^{\infty} \delta^t \left[\sum_{i=1}^{n} u_i(c_i(t), x(t))\right],$$

$$\begin{aligned} \text{where:} \quad & x(0) = \varkappa, \\ & x(t+1) = F\left(x(t) - \sum_{i=1}^{n} c_i(t)\right), \quad t = 0, 1, 2, ..., \\ & c_i(t) = \pi_i(x(t)), \quad i = 1, ..., n, \quad t = 0, 1, 2, ... \end{aligned}$$

A policy $\boldsymbol{\pi}^{\circ}$ is optimal if it is feasible and if, for all $\varkappa$:

$$W\left(\boldsymbol{\pi}^{\circ}, \varkappa\right) \geq W\left(\boldsymbol{\pi}, \varkappa\right),$$

where $\boldsymbol{\pi}$ is any feasible policy.

In order to derive the two next results, we use the following specification of the general model:

$$\begin{aligned} F(x) &= x^{\alpha}, 0 < \alpha \leq 1, \\ u_i(c_i, x) &= \ln(c_i). \end{aligned}$$

It was initially introduced by Levhari and Mirman (1980).

Under the specification of Levhari and Mirman (1980), we have the following two propositions.

Proposition 1. The strategic profile $\mathbf{s}^* = (s_i^*)_{i=1}^n$, where:

$$s_i^*(x) = (1/n)(1-\beta)x, \text{ for all } i \text{ and all } x,$$

is a SMNE, where:

$$0 < \beta \equiv \alpha\delta / (n(1-\alpha\delta) + \alpha\delta) < 1.$$

Proposition 2. The policy $\boldsymbol{\pi}^\circ = (\pi_i{}^\circ)_{i=1}^n$, where:

$$\pi_i{}^\circ(x) = (1/n)(1-\alpha\delta)x, \text{ for all } i \text{ and all } x,$$

is optimal.

From propositions 1 and 2, the global catch rate, in each period, is:

- $(1-\beta)$ % of the current stock, in the unregulated open-access situation,
- $(1-\alpha\delta)$ % of the current stock, in the cooperative situation.

As $\beta \equiv \alpha\delta / \left(n\left(1-\alpha\delta\right)+\alpha\delta\right)$, we have the following table:

Number of countries: / Catch rate under the ...	1	...	∞
Open-access situation:	$1-\alpha\delta$	$\nearrow$	1
Cooperative situation:	...	$1-\alpha\delta$	...

From this:

- The catch rate is larger in the unregulated open-access situation than in the cooperative situation.

In theory, many instruments (entry limitation, licensing, taxes on catches or individual transferable quotas) are capable of restoring efficiency in open-access fisheries (Clark, 1990).

In reality, the data required to determine the optimal policy greatly exceeds the capacity of any resource manager (Arnason, 1990).

Arnason (1990) thus proposes an alternative scheme, where a quota autority:

- allocates Individual and Transferable Share Quotas (ITSQ),
- decides the time path of the Total Allowable Catch (TAC).

Arnason (1990) shows:

- If the ITSQ market is perfectly competitive, the TAC is always fished efficiently,
- Under perfect information and rational expectation, the prevailing ITSQ market price equals the resource rents in the fishery,
- If resource rents and profits are equal, an optimal utilization of the resource is realized simply by adjusting current TAC so as to maximize the market value of ITSQ at each point of time.

The results of Arnason (1990) relies on severe assumptions.
Moreover, Arnason (1990) does not formalize explicitly the TAC adjustment rule.
However, if anticipated by the players, it may generate detrimental incentives.

Below, we follow the same line of research as Arnason's.
The proposed scheme is inspired by the literature on Nash implementation in mechanism design.

A mechanism is denoted (M, g), where:

$M \equiv \times_{i=1}^{n} M_i =$ the message space;

$g : M \longrightarrow \mathbb{R}_{+}^{n} \times \mathbb{R}^{n} =$ the outcome function.

Under a mechanism (M, g), each participant reports a message m_i in M_i. The outcome function converts joint messages $m = (m_i)_{i=1}^{n}$ into consumptions $(C_i(m))_{i=1}^{n}$ and transfers $(T_i(m))_{i=1}^{n}$.

The specific mechanism used below is as follows.
We let $M_i \equiv \mathbb{R}^n \times \mathbb{R}^n_+$, with individual messages denoted $m_i = \left(m_i^C, m_i^P\right) = \left(\left(C_{ik}\right)_{k=1}^n, \left(P_{ik}\right)_{k=1}^n\right)$.

One should interpret the component of a message as follows:

C_{ik} = an increment that i proposes for k's consumption;
P_{ik} = a compensatory price that i proposes to pay to k;
P_{ii} = a compensatory price that i demands to be paid on other's consumptions.

Agents i's consumption and transfers are:

$$
\begin{aligned}
C_i(m) &= \max\left\{0, \textstyle\sum_{k=1}^{n} C_{ki}\right\}, \\
T_i(m) &= \textstyle\sum_{j\neq i} P_j(m)\, C_i(m) - P_i(m) \textstyle\sum_{j\neq i} C_j(m),
\end{aligned}
$$

where the individualized prices $(P_k(m))_{k=1}^{n}$ are individualized prices, obtained as follows:

1. Rearrange the sequence $(P_{ik})_{i=1}^{n}$ in ascending order.
2. In case where $P_{ik} = P_{jk}$, for some i and j, rearrange in ascending order of indexes.
3. Define agent k's personalized price $P_k(m)$ as the N-th term of the ordered sequence, with $N = n/2$, if n is even, and $N = (n+1)/2$, if n is odd.

Property 1. For all $m \in M$ and all $(c_k)_{k=1}^n \in \mathbb{R}_+^n$, each participant i can report a message m_i' such that:

$$\begin{aligned} \left(C_k\left(m/m_i'\right)\right)_{k=1}^n &= (c_k)_{k=1}^n \\ \left(P_k\left(m/m_i'\right)\right)_{k=1}^n &= \left(P_k\left(m\right)\right)_{k=1}^n \end{aligned}$$

Property 1 means that under the mechanism, each participant is able to decide the consumptions of everyone, without modifying the current system of individualized prices.

Proprerty 2. Assume that $n \geq 3$. Given any $(p_k)_{k=1}^{n} \in \mathbb{R}_{+}^{n}$, let $m \in M$ be any joint message such that $m_i^P = (p_k)_{k=1}^{n}$, for all i. Then, $(P_k(m))_{k=1}^{n} = (P_k(m/m_i'))_{k=1}^{n} = (p_k)_{k=1}^{n}$, for all i and $m_i' \in M_i$.

Property 2 states that, whenever all agents announce the same system of individualized prices, then the mechanism implements it and no unilateral deviation by a single agent can modify it.

Property 3. For all $m \in M$, $\sum_{i=1}^{n} T_i(m) = 0$.

In other words, the mechanism (M, g) is balanced.

Regulated equilibrium

Difference game induced by the mechanism (1)

We suppose now that the fishery is regulated by using repeatedly the mechanism defined above.

With the dynamic of the fish population, this defines a new difference game, in which:

- Each country must report a message m_i in M_i, at each date;
- The time-path of the resource follows from the consumption time-path $\left(C_i\left(m\right)\right)_{i=1}^{n}$.

A (stationary Markovian) strategy of country i is a function:

$$\sigma_i : \mathbb{R}_+ \rightarrow M_i,$$

which associates states x of the fish stock
with messages $m_i = \sigma_i(x)$ of player i.

A strategy profile is denoted $\boldsymbol{\sigma} = (\sigma_i)_{i=1}^n$.

It is said to be feasible if, for all x:

- $C_i(\boldsymbol{\sigma}(x)) \geq 0$, for all i,
- $\sum_{i=1}^n C_i(\boldsymbol{\sigma}(x)) \leq x$.

Given an initial state $\varkappa$ and a (feasible) strategic profile σ, the sum of player i's discounted utility is:

$$J_i(\sigma, \varkappa) = \sum_{t=0}^{\infty} \delta^t \left[u_i(c_i(t), x(t)) - t_i(t) \right],$$

where:

$$x(0) = \varkappa,$$
$$x(t+1) = F\left(x(t) - \sum_{i=1}^{n} c_i(t)\right), \quad t = 0, 1, 2, ...,$$
$$c_i(t) = C_i(\sigma(x(t))), \quad i = 1, ..., n, \quad t = 0, 1, 2, ...,$$
$$t_i(t) = T_i(\sigma(x(t))), \quad i = 1, ..., n, \quad t = 0, 1, 2, ...$$

A strategic profile $\boldsymbol{\sigma}^*$ is said to be a SMNE if it is feasible and if, for all $\varkappa$:

$$J_i\left(\boldsymbol{\sigma}^*, \varkappa\right) \geq J_i\left(\left(\boldsymbol{\sigma}^* / \sigma_i\right), \varkappa\right), \text{ for all } i \text{ and } \sigma_i,$$

where $\left(\boldsymbol{\sigma}^* / \sigma_i\right) = \left(\sigma_1^*, ..., \sigma_{i-1}^*, \sigma_i, \sigma_{i+1}^*, ..., \sigma_n^*\right)$.

Proposition 3. Under the setting of Levhari and Mirman (1980), the strategic profile $\sigma^* = (\sigma_i^*)_{i=1}^n$, where:

$$\sigma_i^*(x) = \left(\left(\frac{(1-\alpha\delta)\,x}{n^2} \right)_{k=1}^{n}, \left(\frac{1}{(1-\alpha\delta)\,x} \right)_{k=1}^{n} \right), \text{ for all } i \text{ and } x,$$

is a SMNE of the difference game induced by (M, g).

Proposition 3 thus ensures the existence of SMNE under some specifications of the model.

Proposition 4. If σ^* is a SMNE, then the policy $\boldsymbol{\pi}^* = (\pi_i^*)_{i=1}^n$, where:

$$\pi_i^*(x) = C_i(\sigma^*(x)), \text{ for all } i \text{ and } x,$$

is an optimal policy.

Proposition 4 implies that a SMNE implements an optimal consumption time path of the resource.
It is true for any preferences $u_i(c_i, x)$ and biological process $F(x)$.

Last slide

Thank you for your attention.