Optimal decentralized management of a natural resource

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Summary. We construct an economic mechanism to realize in Nash equilibrium an optimal consumption time path of a natural resource. For exposition convenience, the analysis is conducted within the model initiated by Levhari and Mirman (1980). This framework allows us to explicitly calculate the consumption time paths of the resource, associated with an open-access regime, with a cooperative management and with a (stationary Markovian) Nash equilibrium of the difference game induced by the proposed mechanism.

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JEL classification. Q20, C73.

1 Introduction.

We build on the model initiated by Levhari and Mirman (1980). We thus consider a fishery which is exploited by several countries. Following the standard analysis, we first compare the unregulated open-access equilibrium with the cooperative solution. We derive the corresponding consumption paths of the fish population and restate a well-known result in the literature, namely, that the unregulated open-access equilibrium leads to overfishing, compared with the cooperative solution.

We then imagine that the fishery is under the juridiction of a regulator, having in mind the European Union's Common Fishery Policy. Many instruments can be used to regulate the fishery, including entry limitation, licensing, taxes on catches or individual transferable quotas. In theory, they all can implement an optimal consumption path of the fish population (Clark, 1990). In reality, the amount of information required to determine the optimal policy renders this approach to fisheries management impractible (Arnason, 1990).

Arnason (1990) provides a original scheme which could overcome this difficulty. It is based on a system of Individual Transferable Share Quota (ITSQ). An ITSQ specifies a given share in the Total Allowable Catch (TAC). Assuming perfect competition and rational expectations, Arnason (1990) gives a condition about the fishing technology, under which the regulator can determine, at each point of time, the optimal TAC, with a minimum information. Formally, the prevailing market price of an ITSQ supplies all relevant information and the optimal TAC should be chosen so as to maximize it, at each point of time.

In this paper, following Arnason (1990), we also supply an economic mechanism capable of implementing an optimal consumption path of a resource, with a minimum information. Under this mechanism, each participant decides both his own consumption and that of the other participants. Individualized prices are set by the participants themselves. In equilibrium, the prevailing price system reflects the participants' expected future rents, at each point of time. Moreover, each participant pays his consumption of the resource at a price equal to the sum of the others' individualized prices. It follows that, in equilibrium, the participants internalize the external opportunity cost of their consumption. Finally, to ensure that the mechanism is balanced, each participant is paid his individualized price, on each unit of the resource consumed by the others.

The proposed economic mechanism improves Arnason (1990) in several directions. First, our implementation result is true, provided that at least three countries fish the water, whereas Arnason (1990) relies on the assumption of a perfectly competitive ITSQ market. Second, our implementation result can be shown to be true under a large class of fishing technologies, whereas Arnason (1990) is restricted to fishing technologies such that resource rents and profits are equal within operating fishers (assumption 2). Third, under the mechanism we propose, the regulator's role is limited to enforcing the mechanism, whereas Arnason (1990) requires that the quota authority adjusts the TAC, at each point of time, so as to maximize the market value of ITSQ, but is silent about the procedure to be used.

To emphasize our contribution to the literature, it seems useful to say that this paper stands at the junction between natural resource economics and implementation theory (see Jackson, 2001, for a recent survey). Precisely, we use methods from the mechanism design literature to manage a dynamic externality. We thus provide an implementation result in a difference game. To the best of our knowledge, this has never been done.

The rest of the paper is organized as follows. Section 2 sets the economic model, based on Levhari and Mirman (1980). We derive and compare the consumption paths arising in the situation of unregulated open-access to the fishery and in the situation where the countries cooperate. In section 3, we construct our economic mechanism to regulate the fishery and derive some of its properties. In section 4, we study the set of Nash equilibria of the associated difference game and show our implementation result. Section 5 concludes and discusses several generalizations.

2 The model.

Consider the standard model of fishery, as initiated by Levhari and Mirman (1980). Let $x(t)$ be the quantity of fish at time t. Suppose that, if uniterrupted, the quantity of fish would grow according to the biological rule:

$$x(t+1) = x(t)^{\alpha}, \quad 0 < \alpha \leq 1.$$

Thus one easily observes that $x(t) = 1$, for all t, is a steady state of the fish population.

Suppose, however, that n countries fish the waters. Let c_i be the present consumption of country i and assume that country i has a utility function $u_i = \ln(c_i)$ for present consumption. Let $0 < \delta < 1$ be the common discount factor. Suppose, moreover, that the objective of each country is to maximize the sum of the discounted utility of fish.

A consumption path $\boldsymbol{C}$ gives a consumption $c_i(t)$, for all i and all t. It is said to be feasible if:

$$\begin{array}{l} x(0) = x_0, \\ x(t+1) = \left[x(t) - \sum_{i=1}^{n} c_i(t)\right]^{\alpha}, \quad t = 0,1,2,..., \\ c_i(t) \geq 0, \quad i = 1,...,n, \quad t = 0,1,2,..., \\ \sum_{i=1}^{n} c_i(t) \leq x(t), \quad t = 0,1,2,..., \end{array}$$

where x_0 is the initial state of the fish population.

2.1 The unregulated open-access fishery.

The behavior of the countries under open-access is analysed here. This situation is formalized as a difference game (Clemhout and Wan, 1979), i.e., a discrete-time analog of a differential game (Levhari et Mirman, 1980). Each country has an interest in the long-run effect of its present catch. Moreover, each country must take the catch of the others countries into consideration when deciding on his own catch. The former consideration is accounted for by using a dynamic programming argument, and the latter by using the concept of Nash equilibrium. Precisely, following Levhari et Mirman(1980), we compute a particular subgame-perfect equilibrium in which players' strategies depend only on the current stock of fish (called stationary Markovian strategies). For the sake of completeness, we explicitly derive all results, although they can be found in Kwon (2006).

A (stationary Markovian) strategy of country i is a function $s_i : \mathbb{R}_+ \to \mathbb{R}_+$, which associates states x of the fish stock with consumptions $c_i = s_i(x)$ of player i. A strategic profile is a vector $\boldsymbol{s} = (s_i)_{i=1}^n$. It is said to be feasible if it satisfies, for all x, $s_i(x) \geq 0$, for all i, and $\sum_{i=1}^n s_i(x) \leq x$ [1].

A (feasible) strategic profile $\boldsymbol{s}$ induces a unique consumption path $\boldsymbol{C}$, as follows. The initial state is $x(0) = x_0$. In period 0, countries' consumptions are $c_i(0) = s_i(x(0))$, for all i. Then, at time $t = 1$, the state is $x(1) = \left[x(0) - \sum_{i=1}^n c_i(0)\right]^\alpha$. From this, in period 1, countries consumptions are $c_i(1) = s_i(x(1))$, for all i. And so on...

Given any initial state $\varkappa$, let $w_i(\boldsymbol{s}, \varkappa)$ be the sum of player i's discounted utility, along the consumption path $\boldsymbol{C}$, induced by the strategic profile $\boldsymbol{s}$:

$$w_i(\boldsymbol{s}, \varkappa) = \sum_{t=0}^{\infty} \delta^t \ln(c_i(t)), \tag{1}$$

$$\text{where:} \quad \begin{aligned} & x(0) = \varkappa, \\ & x(t+1) = \left[x(t) - \textstyle\sum_{i=1}^n c_i(t)\right]^\alpha, \quad t = 0, 1, 2, ..., \\ & c_i(t) = s_i(x(t)), \quad i = 1, ..., n, \quad t = 0, 1, 2, ... \end{aligned}$$

A strategic profile $\boldsymbol{s}^*$ is said to be a (stationary Markovian) Nash equilibrium if it is feasible and if, for all $\varkappa$:

$$w_i(\boldsymbol{s}^*, \varkappa) \geq w_i((\boldsymbol{s}^*/s_i), \varkappa), \text{ for all } i \text{ and } s_i,$$

where $(\boldsymbol{s}^*/s_i) = \left(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_n^*\right)$ is any (feasible) strategic profile.

Proposition 1. Let $0 < \beta \equiv \alpha\delta / (n(1-\alpha\delta) + \alpha\delta) < 1$. The strategic profile $\boldsymbol{s}^* = (s_i^*)_{i=1}^n$, where, for all i and all x, $s_i^*(x) = (1/n)(1-\beta)x$, is a (stationary Markovian) Nash equilibrium.

Proof. It is clear that $\boldsymbol{s}^*$ is feasible, as $0 < \beta < 1$.

Denote $x^*(t)$, for all t, the time path of the fish population, induced by the strategic profile $\boldsymbol{s}^*$, given the initial state $\varkappa$ [2]. Let $\lambda(t) = \frac{\alpha}{1-\alpha}(1-\alpha^t)$, for all t. By recurrence, we show that:

$$x^*(t) = \beta^{\lambda(t)} \varkappa^{\alpha^t}, \quad t = 0, 1, 2, ...$$

Indeed, the assumption is true for $t = 0$ (i.e., $x^*(0) = \varkappa$). Assume it is true at time t. By definition of $\boldsymbol{s}^*$, in period t, the total catch is $\sum_{i=1}^n s_i^*(x^*(t)) =$

[1] Feasibility is not a trivial problem in difference games (Clemhout and Wan, 1979). The litterature on fishery largely disregards this problem.

[2] For notation ease, we choose here not to explicitly write $x^*(t)$ as function of $\varkappa$.

$(1-\beta)\, x^*(t)$. It follows that [3]:

$$\begin{aligned} x(t+1) &= \left[x^*(t) - \textstyle\sum_{i=1}^n s_i^*(x^*(t))\right]^\alpha \\ &= \left[\beta^{\lambda(t)+1} \varkappa^{\alpha^t}\right]^\alpha \\ &= \beta^{\lambda(t+1)} \varkappa^{\alpha^{t+1}} \\ &= x^*(t+1), \end{aligned}$$

which proves that the assumption remains true at time $t+1$.

The corresponding consumption path $\boldsymbol{C}^*$ is:

$$c_i^*(t) = s_i^*(x^*(t)) = (1/n)(1-\beta)\, x^*(t), \quad i = 1, ..., n, \quad t = 0, 1, 2, ...$$

Substituting into (1), we obtain:

$$\begin{aligned} w_i(\boldsymbol{s}^*, \varkappa) &= \textstyle\sum_{t=0}^\infty \delta^t \ln(c_i^*(t)) \\ &= \frac{(1-\alpha\delta)\ln((1/n)(1-\beta)) + \alpha\delta\ln(\beta) + (1-\delta)\ln(\varkappa)}{(1-\delta)(1-\alpha\delta)}. \end{aligned} \quad (2)$$

For all $\varkappa$, denote $v_i(\varkappa) = w_i(\boldsymbol{s}^*, \varkappa)$. By theorem (Sundaram, 1996, Th. 12.15), the strategy s_i^* is a best-reply of player i if, and only if, for all $\varkappa$, $v_i(\varkappa)$ satisfies the Bellman equation:

$$v_i(\varkappa) = \max_{c_i \in \mathbb{R}_+} \left\{ \ln(c_i) + \delta v_i\left(\left[\varkappa - c_i - \textstyle\sum_{j\neq i} s_j^*(\varkappa)\right]^\alpha\right)\right\}. \quad (3)$$

The derivative of (2) is $\frac{\partial w_i}{\partial \varkappa}(\boldsymbol{s}^*, \varkappa) = v_i'(\varkappa) = \frac{1}{(1-\alpha\delta)\varkappa}$. From this, the RHS of (3) is maximized if, and only if, c_i satisfies the first-order condition:

$$\frac{1}{c_i} - \frac{\alpha\delta}{(1-\alpha\delta)\left(\varkappa - c_i - \sum_{j\neq i} s_j^*(\varkappa)\right)} = 0.$$

Using $s_j^*(\varkappa) = (1/n)(1-\beta)\varkappa$, for all j, we can show that:

$$c_i = (1/n)(1-\beta)\varkappa, \text{ for all } \varkappa.$$

Substituting into the RHS of (3), we finally obtain:

$$\begin{aligned} &\max_{c_i \in \mathbb{R}_+} \left\{ \ln(c_i) + \delta v_i\left(\left[\varkappa - c_i - \textstyle\sum_{j\neq i} \pi_j^*(\varkappa)\right]^\alpha\right)\right\} \\ &= \ln((1/n)(1-\beta)\varkappa) + \delta V_i([\beta\varkappa]^\alpha) \\ &= v_i(\varkappa). \end{aligned}$$

This completes our proof. ■

[3] Note that $\lambda(t+1) = \alpha(\lambda(t)+1)$.

2.2 The optimal policy.

The behavior of the countries assuming a cooperative management of the fishery is analysed here. Precisely, we suppose that a central planner determines each country's catch rate, in order to maximize the discounted sum of all countries' utilities (Levhari et Mirman, 1980). For the sake of completeness, we explicitly derive all results, although they can be found in Kwon (2006).

The social objective is to find a consumption path $\boldsymbol{C}$, in order to maximize the discounted sum of all countries' utilities:

$$\sum_{t=0}^{\infty} \delta^t \left[\sum_{i=1}^{n} \ln\left(c_i\left(t\right)\right)\right].$$

A solution to this problem is said to be optimal.

Anticipating on future needs, it is convenient to derive an optimal consumption path, by means of dynamic programming.

A policy $\boldsymbol{\pi} = \left(\pi_i\right)_{i=1}^{n}$ is a sequence of n functions $\pi_i : \mathbb{R}^+ \to \mathbb{R}^+$, each of which associates states x of the fish stock with country i's consumption $c_i = \pi_i\left(x\right)$. It is said to be feasible if, for all x, $\pi_i\left(x\right) \geq 0$, for all i, and $\sum_{i=1}^{n} \pi_i\left(x\right) \leq x$.

A policy $\boldsymbol{\pi}$ determines a unique consumption path $\boldsymbol{C}$, from any initial state $\varkappa$. Let $W\left(\boldsymbol{\pi}, \varkappa\right)$ be the associated sum of all players' discounted utility:

$$\begin{aligned} & W\left(\boldsymbol{\pi}, \varkappa\right) = \sum_{t=0}^{\infty} \delta^t \left[\sum_{i=1}^{n} \ln\left(c_i\left(t\right)\right)\right], \qquad (4) \\ & \text{where:} \quad x\left(0\right) = \varkappa, \\ & \qquad x\left(t+1\right) = \left[x\left(t\right) - \sum_{i=1}^{n} c_i\left(t\right)\right]^{\alpha}, \quad t = 0,1,2,..., \\ & \qquad c_i\left(t\right) = \pi_i\left(x\left(t\right)\right), \quad i = 1,...,n, \quad t = 0,1,2,.... \end{aligned}$$

A policy $\boldsymbol{\pi}^{\circ}$ is optimal if it is feasible and if, for all $\varkappa$, $W\left(\boldsymbol{\pi}^{\circ}, \varkappa\right) \geq W\left(\boldsymbol{\pi}, \varkappa\right)$, where $\boldsymbol{\pi}$ is any feasible policy.

Proposition 2. The policy $\boldsymbol{\pi}^{\circ} = \left({\pi_i}^{\circ}\right)_{i=1}^{n}$, where, for all i and all x, ${\pi_i}^{\circ}\left(x\right) = \left(1/n\right)\left(1-\alpha\delta\right)x$, is optimal.

Proof. It is clear that $\boldsymbol{\pi}^{\circ}$ is feasible, as $0 < \alpha\delta < 1$.

Denote $x^{\circ}\left(t\right)$, for all t, the time path of the fish population, induced by the policy $\boldsymbol{\pi}^{\circ}$, given the initial state $\varkappa$ [4]. Adapting the proof of proposition 1, we can show that:

$$x^{\circ}\left(t\right) = \left(\alpha\delta\right)^{\lambda(t)} \varkappa^{\alpha^t}, \quad t = 0,1,2,....$$

The associated consumption path $\boldsymbol{C}^{\circ}$ is:

$${c_i}^{\circ}\left(t\right) = {\pi_i}^{\circ}\left(x^{\circ}\left(t\right)\right) = \left(1/n\right)\left(1-\alpha\delta\right)x^{\circ}\left(t\right), \quad i = 1,...,n, \quad t = 0,1,2,....$$

Substituting into (4), we get the value function:

$$\begin{aligned} W\left(\boldsymbol{\pi}^{\circ}, \varkappa\right) &= \sum_{t=0}^{\infty} \delta^t \left[\sum_{i=1}^{n} \ln\left({c_i}^{\circ}\left(t\right)\right)\right] \\ &= n\left[\frac{\left(1-\alpha\delta\right)\ln\left(\left(1/n\right)\left(1-\alpha\delta\right)\right) + \alpha\delta\ln\left(\alpha\delta\right) + \left(1-\delta\right)\ln\left(\varkappa\right)}{\left(1-\delta\right)\left(1-\alpha\delta\right)}\right] \qquad (5) \end{aligned}$$

[4] For the sake of notation ease, we do not explicitly write $x^{\circ}\left(t\right)$ as function of $\varkappa$.

For all $\varkappa$, denote $V(\varkappa) = W(\boldsymbol{\pi}^\circ, \varkappa)$. By theorem (Sundaram, 1996, Th. 12.15), the policy $\boldsymbol{\pi}^\circ$ is optimal if, and only if, for all $\varkappa$, $V(\varkappa)$ satisfies the Bellman equation:

$$V(\varkappa) = \max_{(c_i)_{i=1}^n \in \mathbb{R}_+^n} \left\{ \sum_{i=1}^n \ln(c_i) + \delta V\left(\left[\varkappa - \sum_{i=1}^n c_i\right]^\alpha \right) \right\}, \tag{6}$$
$$\text{subject to: } \sum_{i=1}^n c_i \leq \varkappa.$$

The derivative of (5) is $\frac{\partial}{\partial \varkappa} W(\boldsymbol{\pi}^\circ, \varkappa) = V'(\varkappa) = \frac{n}{(1-\alpha\delta)\varkappa}$. Thus, the RHS of (6) is maximized if, and only if, $(c_i)_{i=1}^n$ satisfies the first-order conditions:

$$\frac{1}{c_i} - \frac{\alpha\delta n}{(1-\alpha\delta)\left(\varkappa - \sum_{i=1}^n c_i\right)} = 0, \quad i = 1, ..., n,$$

which imply:

$$(c_i)_{i=1}^n = ((1/n)(1-\alpha\delta)\varkappa)_{i=1}^n, \text{ for all } \varkappa.$$

Substituting into the RHS of (6), we finally obtain:

$$\begin{aligned}
&\max_{(c_i)_{i=1}^n} \left\{ \sum_{i=1}^n \ln(c_i) + \delta V\left(\left[\varkappa - \sum_{i=1}^n c_i\right]^\alpha \right) \right\}, \\
&= \sum_{i=1}^n \ln((1/n)(1-\alpha\delta)\varkappa) + \delta V([\alpha\delta\varkappa]^\alpha), \\
&= V(\varkappa).
\end{aligned}$$

This proves the proposition. ■

2.3 Social consequences of open-access.

As a corollary of propositions 1 and 2, we verify a well-known result in the litterature (Gordon, 1954; Levhari and Mirman, 1980), namely that the situation of unregulated open-access to the fishery leads to overfishing, as compared with the optimal policy.

Indeed, from proposition 1, under open-access, all countries catch $(1-\beta)$ % of the current stock of the resource, in each period, with $\beta = \alpha\delta / (n(1-\alpha\delta) + \alpha\delta)$. From proposition 2, it is optimal to harvest $(1-\alpha\delta)$ % of the current stock of the resource, in each period. As β is decreasing in n and is equal to $\alpha\delta$ when $n = 1$, the open-access situation always induces an excessive catch rate, as compared with the optimum. Moreover, overfishing increases in the number of countries.

3 Fishery regulation.

In this section,we imagine that the fishery is under the juridiction of a regulator, having in mind the European Union's Common Fisheries Policy. Following Arnason (1990), we argue that the huge information required to manage fisheries optimally, by means of standard regulatory instruments, appeals for alternative management schemes. We review one such scheme, due to Arnason (1990), and discuss its weacknesses. We then provide a new management scheme, which consists in an economic mechanism, to be used repeatedly to regulate the fishery. We state its properties and define the (stationary Markovian) Nash equilibria of the associated difference game.

3.1 Minimum information management schemes.

Many instruments are available to regulate fisheries, including entry limitation, licensing, taxes on catches or individual transferable quotas. In principle, given certain conditions, all these management measures can be shown to be capable of restoring efficiency in open-access fisheries (Clark, 1990).

In reality, the data required to determine the optimal policy greatly exceeds the capacity of any resource manager, rendering that approach to the fisheries management problem impracticable in most cases (Arnason, 1990). Moreover, arguing that all information is already available within the fishing industry, Arnason (1990) also explains that most of the work necessary for the authority to determine the optimal policy will merely constitute a duplication of work already carried out by private agents in the fishery.

Arnason (1990) thus proposes the following alternative scheme. A quota autority initially allocates Individual and Transferable Share Quotas (ITSQ) and decides the Total Allowable Catch (TAC) at each point of time. A share quota allows the holder the stated share in the TAC in perpetuity.

Arnason (1990) shows this framework can realize an optimal utilization of the resource with minimum information. If the market for ITSQ is perfectly competitive, the TAC will always be fished in the most efficient manner (lemma 1). Thus, the quota authority can ensure optimal utilization of the fish stock by selecting the appropriate time path of TAC. Moreover, if the participants to the market have perfect information and formulate rational expectations about the current and future conditions in the fishery (assumption 1), the prevailing ITSQ market price is equal to the present value of expected future rents generated in the fishery (proposition 1). Thus, under an assumption about technologies, such that resource rents and profits are equivalent within operating fishers (assumption 2), the quota authority can ensure optimal utilization of the resource by adjusting current TAC so as to maximize the market value of ITSQ at each point of time (proposition 2).

However, the system of share quotas proposed by Arnason (1990) has several shortcomings. First, proposition 1 relies on the assumption of a perfectly competitive market for ITSQ. Although Arnason (1990) does not discuss the converse assumption, one can easily foresee that proposition 1 may fail in this case. Indeed, if the participants anticipate that the quota authority sets the TAC depending on the prevailing market price of an ITSQ, they have an incentive to manipulate it. Thus, the equilibrium price may not reveal the present value of expected future rents generated in the fishery. Second, proposition 2 may also not be true if assumptions 1 and 2 above do not hold. Arnason (1990) himself discusses this issue. He argues, however, that condition 1 is natural if one seeks for a market-based management and condition 2 is not that restrictive.

Third, Arnason (1990) is silent about the procedure the quota authority could adopt to adjust the TAC, at each point of time, so as to maximize the market value of ITSQ. Typically, one imagines some iterative procedure which, under the assumption of immediate adjustement of the market to an equilibrium (Arnason, 1990), is presumed to converge to the relevant TAC. However, Arnason should have described more precisely the kind of procedure he has in mind.

3.2 The new mechanism.

A mechanism is a pair (M, g), consisting of a message space $M \equiv \times_{i=1}^{n} M_i$ and an outcome function g. Under the mechanism, each participant i is asked to announce a message m_i in M_i. The outcome function g is a mapping from M into $\mathbb{R}_+^n \times \mathbb{R}^n$, which translates joint messages $m = (m_i)_{i=1}^n$ into consumptions $(C_i(m))_{i=1}^n$ and transfers $(T_i(m))_{i=1}^n$ to be implemented by the participants.

The specific mechanism used below is as follows.

We let $M_i \equiv \mathbb{R}^n \times \mathbb{R}_+^n$, for all i. A generic message of agent i is denoted $m_i = \left(m_i^C, m_i^P\right) = ((C_{ik})_{k=1}^n, (P_{ik})_{k=1}^n)$.

The component C_{ik} is interpreted as an increment of agent k's fish consumption that agent i is willing. (A negative C_{ik} means agent i wants agent k to reduce his consumption by an amount $-C_{ik}$.) Likewise, the component C_{ii} is an increment that agent i is willing for his own fish consumption. The component P_{ik} is a compensatory price that agent i is proposing to pay to agent k per fish consumed by himself. Finally, P_{ii} is a compensatory price that agent i is willing to receive per fish consumed by the other agents.

Agent i's consumption is given by:

$$C_i(m) = \max\left\{0, \textstyle\sum_{k=1}^n C_{ki}\right\}. \tag{7}$$

In order to obtain the transfer to be paid by agent i, several steps are needed.

To begin with, for all k, rearrange the sequence $(P_{ik})_{i=1}^n$ in ascending order. In case where $P_{ik} = P_{jk}$, for some i and j, rearrange in ascending order of indexes. Then, define the agent k's personalized price $P_k(m)$ as the N-th term of the ordered sequence, with $N = n/2$, if n is even, and $N = (n+1)/2$, if n is odd.

Finally, agent i's transfer is given by:

$$T_i(m) = \textstyle\sum_{j \neq i} P_j(m)\, C_i(m) - P_i(m) \sum_{j \neq i} C_j(m). \tag{8}$$

The following properties of the mechanism will prove to be useful below.

Property 1. For all $m \in M$ and all $(c_k)_{k=1}^n \in \mathbb{R}_+^n$, each participant i can report a message m_i' such that $(C_k(m/m_i'))_{k=1}^n = (c_k)_{k=1}^n$ and $(P_k(m/m_i'))_{k=1}^n = (P_k(m))_{k=1}^n$.

Property 1 means that under the mechanism, each participant is able to decide the consumptions of everyone, without modifying the current system of individualized prices.

Proof. Pick $m \in M$ and $(c_k)_{k=1}^n \in \mathbb{R}_+^n$. Consider any agent i. Let $m_i' = \left((C_{ik}')_{k=1}^n, (P_{ik}')_{k=1}^n\right)$ be such that, for all k, $C_{ik}' = c_k - \sum_{j\neq i} C_{jk}$ and $P_{ik}' = P_{ik}$. It is immediate that $C_k(m/m_i') = \max\left\{0, C_{ik}' + \sum_{j\neq i} C_{jk}\right\} = c_k$ and $P_k(m/m_i') = P_k(m)$, for all k. ■

Proprerty 2. Assume that $n \geq 3$. Given any $(p_k)_{k=1}^n \in \mathbb{R}_+^n$, let $m \in M$ be any joint message such that $m_i^P = (p_k)_{k=1}^n$, for all i. Then, $(P_k(m))_{k=1}^n = (P_k(m/m_i'))_{k=1}^n = (p_k)_{k=1}^n$, for all i and $m_i' \in M_i$.

Property 2 states that, whenever all agents announce the same system of individualized prices, then the mechanism implements it and no unilateral deviation by a single agent can modify it. (It is equivalent to say that, whenever all agents but one report the same price system, then the mechanism enforces it.)

Proof. Let $(p_k)_{k=1}^n \in \mathbb{R}_+^n$. Let $m \in M$ be such that $m_i^p = (P_{ik})_{k=1}^n = (p_k)_{k=1}^n$, for all i.

By definition, for all k, $P_k(m)$ is the N-th term of the sequence $(P_{ik})_{i=1}^n$, rearranged in ascending order of values, and then of indexes. Since $(P_{ik})_{i=1}^n = (p_k, ..., p_k)$, we have $P_k(m) = p_k$.

Now, consider any i and $m_i' \in M_i$. Denote $m_i^{p\prime} = (P_{ik}')_{k=1}^n$ the associated vector of personalized prices announced by i. By definition, for all k, $P_k(m/m_i')$ is the N-th term of the sequence $\left(P_{1k}, ..., P_{(i-1)k}, P_{ik}', P_{(i+1)k} ..., P_{nk}\right)$, rearranged in ascending order of values, and then of indexes. The ordered sequence is:

$$\begin{aligned} (P_{ik}', p_k, ..., p_k), \text{ if } P_{ik}' &< p_k, \\ (p_k, ..., p_k), \text{ if } P_{ik}' &= p_k, \\ (p_k, ..., p_k, P_{ik}'), \text{ if } p_k &< P_{ik}'. \end{aligned}$$

In any case, given that $n \geq 3$, we obtain $P_k(m/m_i') = p_k$. ■

Property 3. For all $m \in M$, $\sum_{i=1}^n T_i(m) = 0$.

In other words, the mechanism (M, g) is balanced.

Proof. For all $m \in M$, notice that the transfer $T_i(m)$ can also be written as:

$$T_i(m) = \sum_{j=1}^n P_j(m) C_i(m) - P_i(m) \sum_{j=1}^n C_j(m).$$

Summing over i, one directly obtains:

$$\sum_{i=1}^n T_i(m) = 0,$$

proving that the mechanism (M, g) is balanced. ■

Remark 1. As defined above, the mechanism (M, g) is not always feasible. Precisely, given a current state x of the fish population, there exists m in M

such that $\sum_{i=1}^{n} C_i(m) > x$. However, if we assume that the regulator observes the current stock, this problem can be dealt with, provided that the consumption vector $(C_i(m))_{i=1}^{k}$ is defined as the solution to the following minimization problem:

$$\begin{array}{ll} \min\limits_{(C_i)_{i=1}^{n}} & \sum_{i=1}^{n} \left(C_i - \sum_{k=1}^{n} C_{ki}\right)^2 \\ \text{subject to:} & C_i \geq 0, \quad i = 1, ..., n, \\ & \sum_{i=1}^{n} C_i \leq x. \end{array}$$

Note that the results below would still be true under this amended definition, with slight modifications in our proofs. However, this would request a more active role of the regulator, who must then observe, at each point of time, the current state of the resource.

3.3 Regulated equilibrium.

Suppose that the fishery is regulated by using repeatedly the mechanism defined above. With the dynamic of the fish population, this defines a difference game (Clemhout and Wan, 1979). We formally define here the (stationary Markovian) Nash equilibria of this difference game.

Consider the difference game induced by (M, g).

A (stationary Markovian) strategy of country i is a function $\sigma_i : \mathbb{R}_+ \to M_i$, which associates states x of the fish stock with messages $m_i = \sigma_i(x)$ of player i. A strategy profile is denoted $\boldsymbol{\sigma} = (\sigma_i)_{i=1}^{n}$. It is said to be feasible if, for all x, $C_i(\boldsymbol{\sigma}(x)) \geq 0$, for all i, and $\sum_{i=1}^{n} C_i(\boldsymbol{\sigma}(x)) \leq x$.

A (feasible) strategy profile $\boldsymbol{\sigma}$ determines a unique consumption path $\boldsymbol{C}$, from any initial state $\varkappa$. Let $J_i(\boldsymbol{\sigma}, \varkappa)$ be the associated sum of player i's discounted utility:

$$J_i(\boldsymbol{\sigma}, \varkappa) = \sum_{t=0}^{\infty} \delta^t \left[\ln(c_i(t)) - t_i(t)\right], \tag{9}$$

$$\begin{array}{ll} \text{where:} & x(0) = \varkappa, \\ & x(t+1) = \left[x(t) - \sum_{i=1}^{n} c_i(t)\right]^{\alpha}, \quad t = 0, 1, 2, ..., \\ & c_i(t) = C_i(\boldsymbol{\sigma}(x(t))), \quad i = 1, ..., n, \quad t = 0, 1, 2, ..., \\ & t_i(t) = T_i(\boldsymbol{\sigma}(x(t))), \quad i = 1, ..., n, \quad t = 0, 1, 2, ... \end{array}$$

A strategic profile $\boldsymbol{\sigma}^*$ is said to be a (stationary Markovian) Nash equilibrium if it is feasible and if, for all $\varkappa$:

$$J_i(\boldsymbol{\sigma}^*, \varkappa) \geq J_i((\boldsymbol{\sigma}^*/\sigma_i), \varkappa), \text{ for all } i \text{ and } \sigma_i,$$

where $(\boldsymbol{\sigma}^*/\sigma_i) = \left(\sigma_1^*, ..., \sigma_{i-1}^*, \sigma_i, \sigma_{i+1}^*, ..., \sigma_n^*\right)$ is any (feasible) strategic profile.

4 Implementation result.

We analyse here the properties of (stationary markovian) Nash equilibriums of the difference game induced by the mechanism (M, g). Within the framework of Levhari and Mirman (1980), we first show that the difference game admits at least one (stationary markovian) Nash equilibrium (proposition 3). We then prove that any (stationary markovian) Nash equilibrium realizes an optimal consumption of the resource (proposition 4). We finally argue that proposition 4 is valid in a general setting (remark 2).

Proposition 3. Assume $n \geq 3$. The strategic profile $\boldsymbol{\sigma}^* = (\sigma_i^*)_{i=1}^n$, where, for all i and x:

$$\sigma_i^*(x) = \left(\left(\frac{(1-\alpha\delta)\,x}{n^2} \right)_{k=1}^n, \left(\frac{1}{(1-\alpha\delta)\,x} \right)_{k=1}^n \right),$$

is a (stationary Markovian) Nash equilibrium of the difference game induced by (M, g).

Proof. Consider the strategic profile $\boldsymbol{\sigma}^* = (\sigma_i^*)_{i=1}^n$, where, for all i and x:

$$\sigma_i^*(x) = \left(\left(\frac{(1-\alpha\delta)\,x}{n^2} \right)_{k=1}^n, \left(\frac{1}{(1-\alpha\delta)\,x} \right)_{k=1}^n \right).$$

By definition of (M, g), we have, for all x:

$$\begin{aligned}
(C_i(\sigma^*(x)))_{i=1}^n &= ((1/n)(1-\alpha\delta)\,x)_{i=1}^n\,, \\
(P_i(\sigma^*(x)))_{i=1}^n &= \left(\frac{1}{(1-\alpha\delta)\,x} \right)_{i=1}^n, \\
(T_i(\sigma^*(x)))_{i=1}^n &= (0)_{i=1}^n\,,
\end{aligned}$$

Notice that, for all x, $(C_i(\sigma^*(x)))_{i=1}^n = \boldsymbol{\pi}^\circ(x)$, where $\boldsymbol{\pi}^\circ$ is the optimal policy given in proposition 2. Therefore, given any initial state $\varkappa$, the strategic profile $\boldsymbol{\sigma}^*$ generated the same consumptions and resource time paths as $\boldsymbol{\pi}^\circ$, which are given by (see the proof of proposition 2):

$$\begin{aligned}
c_i{}^\circ(t) &= C_i(\boldsymbol{\sigma}^*(x^\circ(t))) = (1/n)(1-\alpha\delta)\,x^\circ(t)\,, \quad i = 1, ..., n, \quad t = 0, 1, 2, ..., \\
x^\circ(t) &= (\alpha\delta)^{\lambda(t)}\,\varkappa^{\alpha^t}, \quad t = 0, 1, 2, ...
\end{aligned}$$

Subsituting this into (9), we obtain [5]:

$$\begin{aligned}
J_i(\boldsymbol{\sigma}^*, \varkappa) &= \textstyle\sum_{t=0}^\infty \delta^t \ln(c_i{}^\circ(t)) \\
&= \frac{(1-\alpha\delta)\ln((1/n)(1-\alpha\delta)) + \alpha\delta\ln(\alpha\delta) + (1-\delta)\ln(\varkappa)}{(1-\delta)(1-\alpha\delta)} \qquad (10)
\end{aligned}$$

For all $\varkappa$, denote $v_i(\varkappa) = J_i(\boldsymbol{\sigma}^*, \varkappa)$. By theorem (Sundaram, 1996, Th. 12.15), the strategy σ_i^* is a best-reply of country i if, and only if, for all $\varkappa$,

[5] Remember that $T_i(\sigma^*(x)) = 0$, for all i and x. Thus, we substitute in (9): $t_i(t) = T_i(\sigma^*(x^\circ(t))) = 0$, for all t.

$\upsilon_i(\varkappa)$ satisfies the Bellman equation [6]:

$$\upsilon_i(\varkappa) = \max_{m_i \in M_i} \left\{ \ln\left(C_i\left(m^*/m_i\right)\right) - T_i\left(m^*/m_i\right) + \delta \upsilon_i\left(\left[\varkappa - \sum_{i=1}^n C_i\left(m^*/m_i\right)\right]^\alpha\right) \right\},$$

where:

$$T_i\left(m^*/m_i\right) = \sum_{j \neq i} P_j\left(m^*/m_i\right) C_i\left(m^*/m_i\right) - P_i\left(m^*/m_i\right) \sum_{j \neq i} C_j\left(m^*/m_i\right).$$

Now, from property 1, country i can choose m_i to attain any consumption vector $\left(C_k\left(m^*/m_i\right)\right)_{k=1}^n = \left(c_k\right)_{k=1}^n \in \mathbb{R}_+^n$. Moreover, since $n \geq 3$, from property 2, whatever the unilateral deviation m_i of country i, $\left(P_k\left(m^*/m_i\right)\right)_{k=1}^n = \left(P_k\left(m^*\right)\right)_{k=1}^n$.

Therefore, σ_i^* is a best-reply of i if, and only if, for all $\varkappa$, $\upsilon_i(\varkappa)$ satisfies:

$$\upsilon_i(\varkappa) = \max_{\left(c_k\right)_{k=1}^n \in \mathbb{R}_+^n} \left\{ \begin{array}{c} \ln\left(c_i\right) - \sum_{j \neq i} P_j\left(m^*\right) c_i + P_i\left(m^*\right) \sum_{j \neq i} c_j \\ +\delta \upsilon_i\left(\left[\varkappa - \sum_{i=1}^n c_i\right]^\alpha\right) \end{array} \right\}, \quad (11)$$

where:

$$\left(P_i\left(m^*\right)\right)_{i=1}^n = \left(\frac{1}{\left(1-\alpha\delta\right) x}\right)_{i=1}^n.$$

The derivative of (10) is $\frac{\partial}{\partial \varkappa} J_i\left(\boldsymbol{\sigma}^*, \varkappa\right) = \upsilon_i'(\varkappa) = \frac{1}{(1-\alpha\delta)\varkappa}$. From this, the RHS of (11) is maximized if, and only if, $\left(c_k\right)_{k=1}^n$ satisfies the first-order condition:

$$\begin{aligned} \frac{1}{c_i} - \frac{n-1}{\left(1-\alpha\delta\right) x} - \frac{\alpha\delta}{\left(1-\alpha\delta\right)\left(\varkappa - \sum_{i=1}^n c_i\right)} &= 0, \quad k = i, \\ \frac{1}{\left(1-\alpha\delta\right) x} - \frac{\alpha\delta}{\left(1-\alpha\delta\right)\left(\varkappa - \sum_{i=1}^n c_i\right)} &= 0, \quad k \neq i, \end{aligned}$$

It follows that:

$$\begin{aligned} c_i &= (1/n)\left(1-\alpha\delta\right)\varkappa, \\ \sum_{i=1}^n c_i &= \left(1-\alpha\delta\right)\varkappa. \end{aligned}$$

Substituting into the RHS of (11), we obtain:

$$\begin{aligned} & \max_{\left(c_k\right)_{k=1}^n \in \mathbb{R}_+^n} \left\{ \begin{array}{c} \ln\left(c_i\right) - \sum_{j \neq i} P_j\left(m^*\right) c_i + P_i\left(m^*\right) \sum_{j \neq i} c_j \\ +\delta \upsilon_i\left(\left[\varkappa - \sum_{i=1}^n c_i\right]^\alpha\right) \end{array} \right\} \\ = & \frac{\left(1-\alpha\delta\right)\ln\left((1/n)\left(1-\alpha\delta\right)\right) + \alpha\delta\ln\left(\alpha\delta\right) + \left(1-\delta\right)\ln\left(\varkappa\right)}{\left(1-\delta\right)\left(1-\alpha\delta\right)} \\ = & \upsilon_i(\varkappa) \end{aligned}$$

which proves that the strategy σ_i^* is a best-reply of country i.

[6] Here, we denote $\left(m^*/m_i\right) = \left(\boldsymbol{\sigma}_1^*(\varkappa), ..., \boldsymbol{\sigma}_{i-1}^*(\varkappa), m_i, \boldsymbol{\sigma}_{i+1}^*(\varkappa), ..., \boldsymbol{\sigma}_n^*(\varkappa)\right)$.

Finally, since this is true for all players, the strategy profile $\boldsymbol{\sigma}^*$ is a Nash equilibrium. ■

Proposition 4. If $\boldsymbol{\sigma}^*$ is a (stationary Markovian) Nash equilibrium, then the policy $\boldsymbol{\pi}^* = (\pi_i^*)_{i=1}^n$, where $\pi_i^*(x) = C_i(\boldsymbol{\sigma}^*(x))$, for all i and x, is an optimal policy.

Proof. Let $\boldsymbol{\sigma}^*$ be a (stationary Markovian) Nash equilibrium of the difference game induced by (M, g).

Assume, by way of contradiction, that there exists an initial state $\varkappa$ and a feasible policy $\boldsymbol{\pi}$ such that:

$$W(\boldsymbol{\pi}, \varkappa) > \textstyle\sum_{i=1}^n J_i(\boldsymbol{\sigma}^*, \varkappa). \tag{12}$$

Denote $(c_i(t))_{i=1}^n$ and $x(t)$, for all t, the time paths of the country's consumptions and of the fish population, respectively, associated with the policy $\boldsymbol{\pi}$, starting from the initial state $\varkappa$.

By property 1, used at each point x, each country i can find a strategy σ_i such that, for all x:

$$\begin{aligned}
(C_k((\boldsymbol{\sigma}^*/\sigma_i)(x)))_{k=1}^n &= (\pi_k(x))_{k=1}^n, & (13)\\
(P_k((\boldsymbol{\sigma}^*/\sigma_i)(x)))_{k=1}^n &= (P_k(\boldsymbol{\sigma}^*(x)))_{k=1}^n. & (14)
\end{aligned}$$

From (13), it is clear that the strategic profile $(\boldsymbol{\sigma}^*/\sigma_i)$ implements the same time paths of the country's consumptions and of the fish population as the policy $\boldsymbol{\pi}$. Moreover, from (14), the associated time path of the price system is $(P_k((\boldsymbol{\sigma}^*/\sigma_i)(x(t))))_{k=1}^n = (P_k(\boldsymbol{\sigma}^*(x(t))))_{k=1}^n$, for all t. Thus, we have:

$$J_i((\boldsymbol{\sigma}^*/\sigma_i), \varkappa) = \textstyle\sum_{t=0}^\infty \delta^t [\ln(c_i(t)) - t_i(t)],$$

where, for all t:

$$\begin{aligned}
t_i(t) &= T_i(((\boldsymbol{\sigma}^*/\sigma_i)(x(t)))) \\
&= \textstyle\sum_{j\neq i} P_j(\boldsymbol{\sigma}^*(x(t)))\, c_i(t) - P_i(\boldsymbol{\sigma}^*(x(t))) \sum_{j\neq i} c_j(t).
\end{aligned}$$

Considering the above unilateral deviation σ_i, by each player i in turn, and summing over i, we get:

$$\textstyle\sum_{i=1}^n t_i(t) = 0,$$

and therefore:

$$\begin{aligned}
\textstyle\sum_{i=1}^n J_i((\boldsymbol{\sigma}^*/\sigma_i), \varkappa) &= \textstyle\sum_{t=0}^\infty \delta^t \sum_{i=1}^n [\ln(c_i(t))], & (15)\\
&= W(\boldsymbol{\pi}, \varkappa).
\end{aligned}$$

Now, as $\boldsymbol{\sigma}^*$ is a Nash equilibrium, we have, for all i:

$$J_i(\boldsymbol{\sigma}^*, \varkappa) \geq J_i((\boldsymbol{\sigma}^*/\sigma_i), \varkappa),$$

which implies, by summation over i:

$$\sum_{i=1}^{n} J_i\left(\boldsymbol{\sigma}^*, \varkappa\right) \geq \sum_{i=1}^{n} J_i\left(\left(\boldsymbol{\sigma}^*/\sigma_i\right), \varkappa\right). \tag{16}$$

Together, (15) and (16) imply:

$$\sum_{i=1}^{n} J_i\left(\boldsymbol{\sigma}^*, \varkappa\right) \geq W\left(\boldsymbol{\pi}, \varkappa\right),$$

which contradicts our assumption (12).

Finally, as (M, g) is balanced by property 3 and $\sum_{i=1}^{n} J_i\left(\boldsymbol{\sigma}^*, \varkappa\right) \geq W\left(\boldsymbol{\pi}, \varkappa\right)$, for all initial state $\varkappa$ and feasible policy $\boldsymbol{\pi}$, it follows that the policy $\boldsymbol{\pi}^* = \left(\pi_i^*\right)_{i=1}^{n}$, where $\pi_i^*\left(x\right) = C_i\left(\boldsymbol{\sigma}^*\left(x\right)\right)$, for all i and x, is an optimal policy. ■

Remark 2. Note that the proof of proposition 4 is general. Precisely, it also holds true if: the dynamic of the resource is $x\left(t+1\right) = F\left(x\left(t\right)\right)$, where F is an arbitrary function; and the utility of an agent i is $u_i\left(c_i, x\right)$, where u_i is any function of the agent's consumption and of the fish population. Thus, *provided that a Nash equilibrium of the difference game exists*, the mechanism proposed above realizes in Nash equilibrium an optimal consumption path of the fish population, even in a framework far more general than that of Levhari et Mirman (1980). In this paper, the reason why we restricted our analysis to Levhari et Mirman (1980) is because it permits to calculate explicitely a Nash equilibrium of the difference game (proposition 3), proving in passing its existence. An important extension of this paper will be to construct a proof of existence in the general setting above.

5 Conclusion.

In this paper, a new mechanism has been constructed to regulate the utilization of a natural resource. We obtained the following results. On the one hand, within a simple framework (Levhari and Mirman, 1980), we proved the existence of a (stationary Markovian) Nash equilibrium of the difference game associated by the proposed mechanism (proposition 3). On the other hand, within a general framework, we showed that any (stationary Markovian) Nash equilibrium of the difference game realizes an optimal consumption time path of the resource (proposition 4 and remark 2).

Extensions of this work are expectable in two directions. On the one hand, any attempt to simplify the proposed mechanism would be valuable. In particular, following Rouillon (2008), a close mechanism, with a smaller message space, may be possible, in which each participant simply announces his own catch, the total allowable catch, his individual price and the sum of other players' individual prices. However, we expect that the amended mechanism can only be made *weakly* balanced (Rouillon, 2008). On the other hand, the robustness of the mechanism should be analysed, when dealing with other features of fisheries. In particular, one could wish to account for capital investments or multi-species interelations (Fisher and Mirman, 1996).

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