Cost of public funds, rewards and law enforcement

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Summary. In this paper, we amend the standard model of law enforcement, by introducing a cost of public funds. We derive the associated optimal enforcement policy. We show that, as the cost of public funds increases, the objective of raising public gradually replaces that of deterring harmful activity and that the optimal policy varies accordingly. In particular, we prove that, when the harm caused by the illegal activity is sufficiently small, overdeterrence will be optimal for sufficiently large cost of public funds. We also consider the opportunity for the government to allocate (or to raise additional) public funds to reward the individuals who have not engaged in the illegal activity. We prove that this can be socially worthwhile, provided that the cost of public funds is sufficiently small. The reason is that rewarding the legal behaviour enhances the deterrence resulting from any enforcement policy and, thus, allows the government to economize on enforcement expenditures.

1. Introduction.

The literature on law enforcement from which this article derives is based on two basic insights. One is due to Becker (1968). He argues that as any particular level of deterrence can be achieved with different combinations of the fine and the probability of detection, society should employ the highest possible fine and a correspondingly low probability of detection in order to economize on enforcement expenditures. The other is due to Polinsky and Shavell (1984). They explain that if all harmful activities are deterred, decreasing the probability of detection and conviction has no significant detrimental effect on social welfare, although more individuals commit the harmful act, but this saves on the enforcement expenditures. In other words, the expected sanction should be less than the harm borne by victims and some underdeterrence is socially optimal.

A huge literature deals with the robustness of these conclusions, amending the standard model of law enforcement. It is out of the scope of this paper to review it (see, for example, Polinsky and Shavell, 2000). In place, we focus on those articles which are closely related to the topics we deal with below.

Most of the literature on law enforcement rests on an implicit assumption, namely that the government can raise public funds at no cost. To the best of our knowledge, only Garoupa and Jellal (2002) envisage the converse assumption, by introducing in the usual model of law enforcement a cost of public funds (i.e., a cost of transferring one unit of money from the taxpayers to the government). This assumption is common in the literature dealing with the regulation of monopoly under asymmetric information (Laffont and Tirole, 1993). It is justified if only distortionary schemes, such as capital, income or good taxes, are available to the government to raise additional public funds. Garoupa and Jellal (2002a) show (proposition 1) that a positive cost of public funds augments the degree of underdeterrence at an optimum, by increasing the cost of the enforcement policy ([2]). Moreover, if the enforcement agency has private information about the enforcement technology, the cost of the enforcement policy also

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([2]) Including the cost of enforcement itself, plus the deadweight losses incurred to finance it through taxation.

includes informational rents and the degree of underdeterrence should be even larger (proposition 2). To conclude, their results reinforce the traditional belief that some underdeterrence is socially optimal.

This paper builds on Garoupa and Jellal (2002). That is, we also propose to amend the standard law enforcement model, assuming the existence of a positive cost of public funds. Our model differs from their in one respect. In computing the optimal enforcement policy, Garoupa and Jellal (2002a) suppose implicitly that the policy-maker disregards the amount and/or the allocation of fines collected. Formally, this translates into their model by the fact that the social welfare is constant when the fine revenue varies, all other things being equal. However, if the fines paid by offenders are allocated either to the enforcement agency or to the government, we are driven to a contradiction. Indeed, in any case, an increase in the fine revenue allows the government to diminish other taxes by the same amount, inducing an indirect social gain whenever the cost of public funds is positive ([3]). From this, when the government recovers ultimately the fine revenue, the public funds necessary to finance the enforcement policy equal the enforcement expenditures net of the fine revenue and, when the cost of public funds is positive, social welfare is increasing in the fine revenue.

As we can expect, this approach qualitatively modifies the optimal enforcement policy. The policy-maker has now to arbitrate between three objectives, namely reducing the enforcement expenditures, deterring harmful activities and raising fine revenues. To clarify, let us organize our ideas around two polar enforcement policies, corresponding respectively to the situation where the cost of public funds is zero and that where it is arbitrarily large (infinite). The first assumption is the usual setting in the literature. From this, at an optimum, the policy-maker seeks to deter all harmful acts whose net social damage offsets the marginal cost of enforcement (Polinsky and Shavell, 2000). As a consequence, there exists some degree of underdeterrence. With an arbitrarily large cost of public funds, the policy-maker's objective is to maximize the net revenue of the enforcement policy, that is, the difference between the (expected) fine revenue and the enforcement expenditures. Thus, at an optimum, he behaves like a monopolistic enforcer (Polinsky, 1980). It can be shown that the associated enforcement policy can be more or less stringent (in terms of deterrence) than the one discussed previously (Garoupa, 1997). Now, intuitively, as the cost of public funds increases, the objective of raising public funds gradually replaces that of deterring harmful activities. Thus, one expects the optimal enforcement policy to move away from the first polar enforcement policy above and to get closer to the second one. We show below that this insight is correct. Precisely, our results state that: 1) The optimal fine is always maximal, whatever the cost of public funds; 2) The optimal probability of detection and conviction is monotone in the cost of public funds, varying from its values under the first and the second polar enforcement policies ([4]). We also derive a notable consequence of our results, that some degree of overdeterrence is socially worthwhile, provided that the harm is small enough and the cost of public funds is large enough (corollary 1). This occurs when it is socially cheaper to raise additional public funds by means of the enforcement policy than with the traditional fiscal instruments. This result contradicts the standard literature on law enforcement. More importantly (since the two settings are very close), this goes against Garoupa and Jellal (2002a), who prove that a positive cost of public funds can only worsen the degree of underdeterrence.

In this paper, we also depart from the literature by considering the opportunity to allocate (or to raise additional) public funds to reward the individuals who have not engaged in a harmful activity. This idea comes immediately to mind in our setting, where the government arbitrates

([3]) When the fines are allocated to the enforcer, an increase in the fine revenue allows to diminish the subsidy and has the same final effect, thought indirectly.

([4]) In fact, we give more precise results. In particular, we show that the optimal probability is increasing in the cost of public funds for small level of harm, and conversely. See proposition 1.

between the cost, the revenue and the deterrent effect of the enforcement policy. Indeed, rewarding the individuals who did not commit the harmful act has formally the same effect as increasing the fine. Therefore, this instrument would allow the government to economize on enforcement expenditures, all other things being equal (in particular, with the same deterrence effect). Intuitively, this will be socially beneficial if the cost of public funds is small enough. Just as we argued above that the government could be tempted to use the enforcement policy as a way to raise public funds, symmetrically, he could also consider the possibility of using public funds to reward the individuals who choose the legal behaviour, in order to save on enforcement expenditures. To the best of our knowledge, this idea has never been explored in the literature. We show in this paper that it is absolutely valid. No matter the enforcement problem in consideration, we prove that it is socially beneficial to reward the individuals who do not commit the harmful act provided the cost of public funds is sufficiently small.

The discussion is organized as follows. Section 2 sets out the basic model of law enforcement, assuming a positive cost of public funds, and gives the associated optimal enforcement policy. Section 3 carries on the model, by introducing the possibility of rewarding individuals who did not commit the harmful act, and expounds the corresponding optimal enforcement policy. Section 4 concludes. Most proofs are relegated in the appendix.

2. The model.

In the model, risk-neutral individuals contemplate whether to commit an act that yields benefits b to them and harms the rest of society by h. The policy-maker observes the harm h, not the individual's benefit b. However, he knows the distribution of b among the population, described by a general density function $g(b)$, with support $[0, \infty)$, a cumulative distribution $G(b)$, and a distribution elasticity $\varepsilon(b) = -\, b\, g'(b)/g(b)$.

The government sets the enforcement policy, by choosing a fine, f, and a probability of detection and conviction, p. The expenditure on detection and conviction to achieve a probability p is given by $c(p)$, where $c'(p) > 0$ and $c''(p) > 0$. The maximum feasible sanction is F, which can be interpreted as the maximum wealth of individuals.

The following assumption will be in force everywhere, in order to eliminate corner solutions.

Assumption 1. $c'(0) = 0$ and $c'(p) \to \infty$ as $p \to 1$.

An individual will commit a harmful act if, and only if, his benefit is greater than or equal to the expected fine

$$b \geq p\, f. \tag{1}$$

The population is normalized so that the total population equals unity. From this, $(1 - G(p\, f))$ individuals will engage in the activity. The government will collect $p\, f\, (1 - G(p\, f))$ as (expected) fine revenue and incurs the enforcement cost $c(p)$. His expected net revenue is thus

$$t = p\, f\, (1 - G(p\, f)) - c(p). \tag{2}$$

In this paper, we depart from the standard model of law enforcement by introducing a cost of public funds. We assume that a one unit increase (resp., decrease) of government spending reduces (resp., increases) the social welfare by λ units. Caillaud and al. (1988) justify the cost of public funds by the use of distortionary taxation for raising funds and/or by invoking an implicit second-best model.

Social welfare is the sum of the benefits individuals obtain from committing harmful acts, less the harms they cause, less the costs of law enforcement, and less the costs of public transfers. Using (2), it can be expressed as

$$S(p, f) = \int_{p f}^{\infty} (b - h)\, g(b)\, \mathrm{d}b + \lambda\, p\, f\, (1 - G(p\, f)) - (1 + \lambda)\, c(p). \tag{3}$$

The social problem is to choose p and f to maximize (3).

Deterring harmful activities.

When the unit cost of public funds is zero (i.e., $\lambda = 0$), the social problem is to choose p and f to deter harmful activities (i.e., activities such that $b < h$), taking into account the cost of enforcement $c(p)$. Since Becker (1968), this question has received an extensive attention in the literature (Polinsky and Shavell (2000) provide a survey). Here, we restate these standard results, denoting $p^*(0)$ and $f^*(0)$ the corresponding optimal solution of (3).

The fine should be set as high as possible. In our framework, $f^*(0) = F$. The optimal probability of detection and conviction $p^*(0)$ should satisfy

$$F\,(h - p^*(0)\, F)\; g(p^*(0)\, F) = c'(p^*(0)). \tag{4}$$

The first term is the deterrent effect of a higher probability of detection, equal to the number of individuals who are deterred, $F\, g(p^*(0)\, F)$, multiplied by their net effect on social welfare, $h - p^*(0)\, F$. The second term is the marginal cost of enforcing a higher probability of detection. As $c'(p) > 0$, this implies that $p^*(0)\, F < h$. Hence, it is always optimal to underdeter harmful activities, in order to save on the enforcement costs. Polinsky (1980) proves that an increase in h leads to a higher optimal probability of detection and conviction $p^*(0)$. On the other hand, the maximal fine F has an ambiguous effect on $p^*(0)$. Precisely, Garoupa (2001) shows that an increase in F leads to a smaller optimal probability $p^*(0)$ if, and only if, $\varepsilon(p^*(0)\, F) \geq 1$ or $p^*(0)\, F$ is sufficiently close to h.

Raising public funds.

When the unit cost of public funds is arbitrarily large (i.e., $\lambda = \infty$), the social problem becomes to choose p and f to maximize the net revenue of the enforcement policy. This problem has not received much attention in the literature ([5]). A notable exception is Garoupa and Klerman (2002). However, they use a linear-quadratic model. Here, we derive the optimal enforcement policy, assuming that the objective is to maximize (2). We denote it $p^*(\infty)$ and $f^*(\infty)$. (This notation anticipates a result given in proposition 1 below, namely that the general solution of problem (3), denoted $p^*(\lambda)$ and $f^*(\lambda)$, for all λ, converges to $p^*(\infty)$ and $f^*(\infty)$, as λ tends to infinity).

Using standard arguments, it is immediate that $f^*(\infty) = F$. Otherwise, if $f^*(\infty) < F$, it would be possible to increase the fine up to $f = F$ and to decrease the probability of detection and conviction p, so that the overall deterrence remains unchanged (i.e., so that $p\, F = p^*(\infty)\, f^*(\infty)$). However, this would decrease the costs of enforcement $c(p)$, while letting the (expected) fine revenue $p\, f\, (1 - G(p\, f))$ unchanged.

Then, the optimal probability of detection and conviction $p^*(\infty)$ should satisfy

$$F\,[1 - G(p^*(\infty)\, F) - p^*(\infty)\, F\, g(p^*(\infty)\, F)] = c'(p^*(\infty)). \tag{5}$$

The first term is the effect of a higher probability of detection and conviction on the (expected) fine revenue. A larger proportion of the $(1 - G(p^*(\infty)\, F))$ individuals who engaged in the harmful activity is detected and thus convicted to pay F. On the other hand, the number of individuals who are deterred increases by $p^*(\infty)\, F\, g(p^*(\infty)\, F)$. The second term is the marginal cost of enforcing a higher probability of detection.

Considering both objectives jointly.

([5]) In fact, Polinsky (1980) and Besanko and Spulber (1989) addressed this question indirectly, when dealing with the delegation of the enforcement activity, either to the public or the private sector.

A robust conclusion in the literature is that it is always socially optimal to underdeter harmful activities, in order to economize on the enforcement costs (Polinsky and Shavell, 2000). We show here that this need not be true, when taking into account the cost of the public funds. Precisely, we prove that, if the harm caused by the harmful activity is sufficiently small and the cost of public funds is sufficiently large, it will be optimal to overdeter harmful activities, in order to increase the net revenue of the enforcement policy. A similar result was obtained recently by Garoupa and Klerman (2002). Two differences should be noted. First, while they use a linear-quadratic model, the one used here is quite general. Second, Garoupa and Klerman (2002) study the behaviour of a self-interested, rent-seeking government. Thus, they derive an "optimal" enforcement policy, which essentially reflects the preferences of the government. On the contrary, in our model, the government is assumed benevolent. Thus, the prescribed policy is optimal in the usual sense. Overdeterrence of harmful activities occurs here only if the government can raise public funds at a lower cost when using the enforcement policy, than when using usual taxation schemes.

Assuming that problem (3) is strictly concave in p, for all λ, we derive the properties of its solution in the following proposition. For all λ, we denote it $p^*(\lambda)$ and $f^*(\lambda)$. The proof is given in the appendix.

Proposition 1. (a) The optimal fine $f^*(\lambda)$ is the maximal fine F. (b) There exists a threshold level for the harm h, denoted $\underline{h}$, such that $p^*(0)$ <, = or > $p^*(\infty)$, whenever h <, = or > $\underline{h}$. (c) The optimal probability of detection and conviction $p^*(\lambda)$ is monotone and varies from $p^*(0)$ to $p^*(\infty)$ as λ goes from zero to infinity.

The reason why the optimal sanction is always maximal is clear, since this simultaneously enables to save on the enforcement costs and to increase the net revenue of the enforcement policy. Turning now to the properties of the optimal probability of detection and conviction, three remarks can be made. First, it should always be chosen between $p^*(0)$ and $p^*(\infty)$ (i.e., the probability that would be optimal if, respectively, the objective was to deter harmful acts only, or to raise public funds only). Second, as the cost of publics funds λ increases, the deterrence objective becomes gradually replaced by the objective of raising public funds and, as a consequence, the optimal probability of detection and conviction moves away from $p^*(0)$ and converges to $p^*(\infty)$. Third, $p^*(\lambda)$ is monotone and, thus, increases (resp., decreases) in λ if $p^*(0) < p^*(\infty)$ (resp., >). In part (b) of the proposition, we provide a condition to discriminate between these two cases. To understand this point, remember that $p^*(0)$ increases in h, while $p^*(\infty)$ does not depend on h. In words, if the regulator seeks to deter harmful activities, he should expense more and more in law enforcement, as the harm caused by an offender gets larger (Polinsky, 1980), while this is unimportant if he only wishes to raise public funds. We show in part (b) that, if the harm is sufficiently small (i.e., $h < \underline{h}$), it will be the case that the objective of raising public funds alone justifies a more stringent enforcement policy than that of deterring harmful activities, in which case $p^*(0) < p^*(\infty)$. Otherwise, if the harm is sufficiently large (i.e., $h > \underline{h}$), the objective of deterring harmful activities alone justifies a more stringent enforcement policy than that of raising public funds, in which case $p^*(0) > p^*(\infty)$.

The corollary below highlights an original consequence of proposition 1, namely that the optimal enforcement policy sometimes implies overdeterrence of harmful activities.

Corollary 1. Let $h° = p^*(\infty)\ F > 0$. If $h < h°$, it will be socially worthwhile to overderdeter harmful activities for sufficiently large λ. (Precisely, there exists $\lambda° > 0$ such that $p^*(\lambda)\ F > h$ if, and only if, $\lambda > \lambda°$). Otherwise, underdeterrence is always optimal.

This property follows directly from the previous discussion. If $p^*(0) \geq p^*(\infty)$, then $p^*(\lambda)$ is constant or decreasing, and $p^*(\lambda)\ F \leq p^*(0)\ F$, for all λ. As $p^*(0)\ F < h$, some underterrence is always optimal. Otherwise (i.e., if $p^*(0) < p^*(\infty)$), then $p^*(\lambda)$ is (strictly) increasing and $p^*(\lambda)\ F$ varies from $p^*(0)\ F$ to $h° = p^*(\infty)\ F$ as λ goes from zero to infinity. Hence, overdeterrence will occur provided that $h < h°$ and that λ is sufficiently large (remembering that $p^*(0)\ F < h$).

4. Rewarding those who do not engage in a harmful activity.

We consider here the possibility of rewarding the individuals who have not engaged in the harmful activity. This questioning is natural in our setting, where the government arbitrates between the cost, the revenue and the deterrent effect of the enforcement policy. Indeed, to reward the individuals who did not commit the harmful act improves the deterrence resulting from any given enforcement cost and fine. As a consequence, this allows the government to save on enforcement costs, which could be socially worthwhile if the cost of public funds is sufficiently small.

A priori, this questioning makes sense only in those areas of law enforcement, where the enforcer audits the individuals at random and convicts them to pay the fine, whenever he finds they have had the wrong behaviour. Examples are the compliance with the traffic code, the tax code or the environmental regulation. Below, we build on the fact that, in these cases, the government could equally ask the enforcer to pay a reward to those individuals that he finds compliant ([6]).

Hence, the model should now be seen as follows. Each individual has a probability p of being visited. After an inspection, the enforcer detects without error whether an individual has engaged in the harmful activity or not ([7]). If he observes the wrong behaviour, he convicts him to pay the fine f. If he finds the expected behaviour, he pays him a reward r. We assume that $0 \leq r \leq R$, where R will be referred to as the maximal reward ([8]).

In this setting, an individual of type b wins $p\ r$, if he does not commit the harmful act, and $b - p\ f$, if he does. As a consequence, he will engage in the harmful activity if, and only if,

$$b \geq p\ (f + r). \qquad (6)$$

It follows directly that the use of a reward enhances deterrence, all other things being equal.

From (6), the government will collect $p\ f\ (1 - G(p\ (f + r)))$ as (expected) fine revenue, will pay $p\ r\ G(p\ (f + r))$ as (expected) rewards, and incurs the enforcement cost $c(p)$. His expected net revenue is thus

$$t = p\ [f\ (1 - G(p\ (f + r))) - r\ G(p\ (f + r))] - c(p). \qquad (7)$$

Social welfare is the sum of the benefits individuals obtain from committing harmful acts, less the harms they cause, less the costs of law enforcement, and less the costs of public transfers. Using (7), it can be expressed as ([9])

([6]) Although we do not pay attention to this possibility, the enforcer could also reward the individuals that he finds compliant by means of a lottery, paying them a given amount with some probability.

([7]) In a general model, we would let Q (resp., q) be the probability of being detected non compliant when this is actually (resp., not) the case, with $q < Q$. Here, we assume that $q = 0$ and $Q = 1$.

([8]) The reward need to be assumed bounded above, since if $R = \infty$, the social problem would have no solution for $\lambda = 0$. Precisely, when $\lambda = 0$, the government can always improve the social welfare by decreasing p and raising r, so that the deterrence outcome remains unchanged.

([9]) Notice that, in section 3, we wrote $S(p, f) = S(p, f, 0)$.

$$S(p, f, r) = \int_{p(f+r)}^{\infty} (b - h)\, g(b)\, db + \lambda\, p\, [f - (f + r)\, G(p\,(f + r))] - (1 + \lambda)\, c(p). \qquad (8)$$

The social problem is to choose p, f and r to maximize (8). Below, we let $p°(\lambda)$, $f°(\lambda)$ and $r°(\lambda)$ denote the optimal solution, for all λ.

Assuming that this problem is strictly concave in r, for all λ, we summarize our results in proposition 3. The proof is relegated in the appendix.

In expounding proposition 3, for all λ, we let $P(\lambda)$ be the solution of the problem of choosing the probability of detection p to maximize $\lambda\, p\, F - (1 + \lambda)\, c(p)$. To interpret, notice that this objective function would coincide with the social welfare if, instead of (6), we had assumed that the individuals were myopic to the enforcement policy and thus, whatever p, f and r, would engage in harmful activity each time $b \geq 0$.

Proposition 2. (a) The optimal fine $f°(\lambda)$ is the maximal fine F. (b) There exists λ_0 and λ_1, with $0 < \lambda_0 < \lambda_1$, such that: (i) If the unit cost of public funds λ is small (i.e., if $\lambda < \lambda_0$), then $p°(\lambda) > P(\lambda)$ and the optimal reward $r°(\lambda)$ is the maximal reward R. (ii) If the unit cost of public funds λ is intermediate (i.e., if $\lambda_0 \leq \lambda \leq \lambda_1$), then $p°(\lambda) = P(\lambda)$ and the optimal reward $r°(\lambda)$ lies between 0 and R. (iii) If the unit cost of public funds λ is large (i.e., if $\lambda > \lambda_1$), then $p°(\lambda) < P(\lambda)$ and the optimal reward $r°(\lambda)$ is the minimal reward 0.

Hence, no matter the enforcement problem in consideration, as defined by F, h and $c(p)$, it is socially optimal to reward the individuals found in compliance, provided that the cost of public funds is small enough (i.e., $\lambda \leq \lambda_1$). The intuition is as follows. Assume that no reward is initially given. Subject to this constraint, the optimal enforcement policy is $p^*(\lambda)$ and $f^*(\lambda) = F$, as stated in proposition 1. From this initial situation, a marginal increase of r yields a variation of the social welfare equal to

$$S_3(p^*(\lambda), F, 0) = p^*(\lambda)\, (h - p^*(\lambda)\, F\, (1 + \lambda))\, g(p^*(\lambda)\, F) - \lambda\, p^*(\lambda)\, G(p^*(\lambda)\, F).$$

The first term is the deterrent effect of the reward, equal to the number of individuals who are deterred, $p^*(\lambda)\, g(p^*(\lambda)\, F)$, multiplied by their net effect on social welfare, $h - p^*(\lambda)\, F\, (1 + \lambda)$ ([10]). The second term is the opportunity cost of the rewards paid, equal to the cost of public funds λ, multiplied by the (expected) amount paid, $p^*(\lambda)\, G(p^*(\lambda)\, F)$. If $\lambda = 0$, it is clear that it can only be socially beneficial to reward the individuals for not engaging in the harmful activity, since this improves deterrence at no cost. Formally, since $p^*(0)\, F < h$, we have $S_3(p^*(0), F, 0) = p^*(0)\, (h - p^*(0)\, F)\, g(p^*(0)\, F) > 0$. Finally, as $p^*(\lambda)$ and $S_3(p, f, r)$ are continuous, $S_3(p^*(\lambda), F, 0)$ is positive over some range $[0, \lambda_1]$.

To go further into this result, it is necessary to characterize the relationship between the threshold λ_1 and the enforcement problem, characterized by F, h and $c(p)$. In order to do this, we generalize the cost function as $C(p, k) = k\, c(p)$, with $k > 0$. ([11]). A larger k means a less efficient technology of law enforcement. Our conclusions are summarized in following proposition. The proof is presented in appendix.

Proposition 3. The threshold level λ_1 of the cost of public funds below which it becomes optimal to reward the individuals found in compliance is decreasing in the maximal fine, and

([10]) Remark that this includes both the net benefit of not committing the act, $h - p^*(\lambda)\, F$, less the opportunity cost of the fines lost, $\lambda\, p^*(\lambda)\, F$.
([11]) Note that the cost function considered above corresponds to the case $k = 1$.

increasing in the harm and the cost of enforcement. (Formally, we prove that $d\lambda_1/dF < 0$, $d\lambda_1/dh > 0$ and $d\lambda_1/dk > 0$.)

This result is very intuitive. In a situation where deterring the harmful acts is highly desirable (i.e., h is large), but very costly to implement (i.e., F is small and $c(p)$ is large), if the government does not reward the individuals found in compliance, the optimal enforcement policy will be far from the first-best optimum. In such a case, rewarding the individuals who choose the desired behaviours is highly profitable, from a social point of view. And if the cost of public funds is small enough, it will be socially worthwhile to do so, even when taking into account the cost of raising the necessary funds.

5. Concluding remarks.

We have dealt with the determination of the optimal enforcement policy by a benevolent regulator who can raise public funds by means of distortionary taxation schemes only. This assumption is translated as a positive cost of public funds in the model. The main results are the following. The enforcement policy can be used either as an instrument to deter socially harmful activities, or as an instrument to raise additional public funds. In general, the two objectives will justify different policies. More precisely, the deterrence objective will induce a more stringent enforcement policy, when the harm caused by the illegal behaviour is large, and conversely. As the cost of public funds increases, the first objective is gradually replaced by the objective of raising public funds, with the corresponding modification of the enforcement policy. In particular, there exist enforcement problems where it can be optimal to overdeter harmful activities, for a sufficiently large cost of public funds. We also analysed the possibility of rewarding individuals who do not engage in a harmful activity. This idea can be applied in any area of law enforcement where the enforcer visits individuals at random, to check whether they complied or not with the law (tax code, traffic code, environmental regulation). We show that it is optimal to reward individuals who do not engage in a harmful activity, if the cost of public funds is sufficiently small. If we relate this result with the parameter of the enforcement policy, we can show that, other things being equal, it will be optimal to reward "good guys" if: the cost of detection and conviction is large; the harm is large; and the maximal fine is small.

Appendix A.

A.1. Proof of proposition 1. For all λ, let $p^*(\lambda)$ and $f^*(\lambda)$ be the optimal solution of (3).

(a) Assume that $f^*(\lambda) < F$. Let p and f be such that $f = F$ and $p\, f = p^*(\lambda)\, f^*(\lambda)$. Hence, $p = p^*(\lambda)\, (f^*(\lambda)/F) < p^*(\lambda)$. We have $S(p, f) - S(p^*(\lambda), f^*(\lambda)) = (1 + \lambda)\, (c(p^*(\lambda)) - c(p)) > 0$.

(b) Let $\underline{h}$ be such that $F\, (\underline{h} - p^*(\infty)\, F)\, g(p^*(\infty)\, F) - c'(p^*(\infty)) = 0$. If $h = \underline{h}$, then (4) and (5) have the same solution (i.e., $p^*(0) = p^*(\infty)$). Finally, remember that $p^*(0)$ is increasing in h (Polinsky, 1980), while $p^*(\infty)$ does not depend on h.

(c) The optimal probability $p^*(\lambda)$ satisfies the first-order condition (using $f^*(\lambda) = F$)

$$[F\, (h - p^*(\lambda)\, F)\, g(p^*(\lambda)\, F) - c'(p^*(\lambda))]$$
$$+ \lambda\, [F\, (1 - G(p^*(\lambda)\, F) - p^*(\lambda)\, F\, g(p^*(\lambda)\, F)) - c'(p^*(\lambda))] = 0. \qquad \text{(A1)}$$

If $S(p, f)$ is strictly concave in p, for all λ, then both terms under brackets in (A1) are decreasing in p. Now, assume that $h < \underline{h}$. For all $\lambda > 0$, the terms under brackets in (A1) must have opposite signs. Here, this implies that $p^*(0) < p^*(\lambda) < p^*(\infty)$, where the first term is negative and the second one positive (otherwise, they have same sign). Moreover, from the implicit function theorem, we show that

$(p^*)'(\lambda) = -[F(1 - G(p^*(\lambda) F) - p^*(\lambda) F \, g(p^*(\lambda) F)) - c'(p^*(\lambda))]/[SOC] > 0,$

where "SOC" refers to the second order condition, which is negative.

If $h > \underline{h}$, the same arguments prove that $p^*(\infty) < p^*(\lambda) < p^*(0)$, and $(p^*)'(\lambda) < 0$.

It is clear that $p^*(\lambda) \to p^*(\infty)$ as $\lambda \to \infty$. Otherwise, there would exist a sequence $\lambda_i \to \infty$ such that $|p^*(\lambda_i) - p^*(\infty)| \geq k$, for some $k > 0$ and all i. The second term under brackets in (A1) would then remain larger, in absolute value, than some $k' > 0$ and, finally, condition (A1) would be violated for sufficiently large λ_i. **QED**

A.2. Proof of corollary 1. Let $h° = p^*(\infty) F$. As in the proof of proposition 1, let $\underline{h}$ be such that $F(\underline{h} - p^*(\infty) F) \, g(p^*(\infty) F) - c'(p^*(\infty)) = 0$. Substituting $h° = p^*(\infty) F$, this implies that $\underline{h} > h°$. If we assume that $h < h°$, then $h < \underline{h}$ and proposition 1 says that $p^*(\lambda)$ is increasing and $p^*(\lambda) \to p^*(\infty)$ as $\lambda \to \infty$. Hence, $p^*(\lambda) F \to h°$ as $\lambda \to \infty$. Since $p^*(0) F < h$, it is then clear that there exists $\lambda° > 0$ such that $p^*(\lambda) F > h$ if, and only if, $\lambda > \lambda°$. Assume now that $h \geq h°$. If $h < \underline{h}$, $p^*(\lambda)$ has the same properties as above. However, $p^*(\lambda) F \to h°$ as $\lambda \to \infty$ and, since $h° \leq h$, it will never reach h. If $h \geq \underline{h}$, proposition 1 says that $p^*(\lambda)$ is non increasing. Thus, for all λ, we have $p^*(\lambda) F \leq p^*(0) F < h$. **QED**

A.3. Proof of proposition 2. For all λ, let $p°(\lambda)$, $f°(\lambda)$ and $r°(\lambda)$ be the optimal solution of problem (8).

(a) It is immediate to prove that $f°(\lambda) = F$, for all λ.

If $p°(\lambda) = 0$, for some λ, whatever f, the social welfare is constant. Hence, $f°(\lambda) = F$ is optimal. Otherwise, assume that $f°(\lambda) < F$, for some λ. We distinguish two cases.

- If $r°(\lambda) = 0$, let p and f be such that $f = F$ and $p \, f = p°(\lambda) \, f°(\lambda)$. Then, $p = p°(\lambda) (f°(\lambda)/F) < p°(\lambda)$ and $S(p, f, 0) - S(p°(\lambda), f°(\lambda), 0) = (1 + \lambda)(c(p°(\lambda)) - c(p)) > 0$.

- If $r°(\lambda) > 0$, let f and r be such that $f°(\lambda) < f \leq F$, $0 \leq r < r°(\lambda)$ and $f + r = f°(\lambda) + r°(\lambda)$. We have $S(p°(\lambda), f, r) - S(p°(\lambda), f°(\lambda), r°(\lambda)) = \lambda \, p°(\lambda)(f - f°(\lambda)) > 0$.

(b) Since $f°(\lambda) = F$, the social problem is equivalent to choose p and $\beta = p(F + r)$ to maximize

$$\int_\beta^\infty (b - h) \, g(b) \, db - \lambda \beta G(\beta) + \lambda p F - (1 + \lambda) c(p), \tag{A2}$$

subject to $p F \leq \beta \leq p(F + R)$.

The lagrangian of this problem is

$$L(\beta, p) = \int_\beta^\infty (b - h) \, g(b) \, db - \lambda \beta G(\beta) + \lambda p F - (1 + \lambda) c(p) - \mu (p F - \beta) - \eta (\beta - p(F + R)),$$

where μ and η are the lagrangian multipliers associated with the inequality constraints.

An optimal solution of problem (A2) satisfies

$$L_1(\beta, p) = \phi(\beta, \lambda) + \mu - \eta = 0, \tag{A3}$$

$$L_2(\beta, p) = \lambda F - (1 + \lambda) c'(p) - \mu F + \eta (F + R) = 0, \tag{A4}$$

$$\mu \geq 0 \text{ and } \mu (p F - \beta) = 0, \tag{A5}$$

$$\eta \geq 0 \text{ and } \eta (\beta - p(F + R)) = 0. \tag{A6}$$

where $\phi(\beta, \lambda) = (h - \beta) \, g(\beta) - \lambda (G(\beta) + \beta g(\beta))$, for all β and λ.

Part (b) of the proposition follows from these conditions. The proof below rests on the assumption that $\phi(\beta, \lambda)$ is (strictly) decreasing in β, for all λ ([12]). In appendix B, we derive a sufficient condition for this and interpret it.

([12]) It is equivalent to assume that (8) is strictly concave in r, for all λ.

Since $\phi(\beta, \lambda)$ is (strictly) decreasing in β, for all λ, $\phi(\beta, \lambda) = 0$ has exactly one solution $\beta = B(\lambda)$, such that $0 < B(\lambda) \leq h$ ([13]). We can show that $B(0) = h$, $B(\lambda) \to 0$ as $\lambda \to \infty$ ([14]) and $B(\lambda)$ is (strictly) decreasing ([15]).

The property below follows directly and will be useful in the end of the proof.

Lemma 1. $\phi(\beta, \lambda)$ >, = or < 0, whenever β <, = or > $B(\lambda)$, respectively.

For all λ, let $P(\lambda)$ satisfy $c'(P(\lambda)) = \lambda F/(1 + \lambda)$. We can show that $P(0) = 0$, $P(\lambda) \to (c')^{-1}(F) > 0$ as $\lambda \to \infty$ and, by the implicit function theorem, $P'(\lambda) = F/[(1 + \lambda)^2 c''(P(\lambda))] > 0$.

There exists λ_0 and λ_1, such that $P(\lambda_0)(F + R) = B(\lambda_0)$ and $P(\lambda_1) F = B(\lambda_1)$ ([16]). Since $P(\lambda)$ is (strictly) increasing and $B(\lambda)$ is (strictly) decreasing, they are unique. It is clear that $0 < \lambda_0 < \lambda_1$.

The property below follows directly and will be useful in the end of the proof.

Lemma 2. a. If $\lambda < \lambda_0$, then $P(\lambda)(F + R) < B(\lambda)$; **b.** If $\lambda_0 \leq \lambda \leq \lambda_1$, then $P(\lambda) F \leq B(\lambda) \leq P(\lambda)(F + R)$; **c.** If $\lambda_1 < \lambda$, then $B(\lambda) < P(\lambda) F$.

We are now in a position to prove parts (i) to (iii).
Denote $\beta°(\lambda)$ and $p°(\lambda)$ the solution of problem (A2).
Three cases can be in force: $\eta > 0$, $\mu = \eta = 0$, or $\mu > 0$, which we deal with in turn below.
(i) Assume that the optimal solution $\beta°(\lambda)$ and $p°(\lambda)$ induces $\eta > 0$. From (A3), $\phi(\beta°(\lambda), \lambda) = \eta > 0$. By lemma 1, this implies that $\beta°(\lambda) < B(\lambda)$. Using (A4), $c'(p°(\lambda)) > \lambda F/(1 + \lambda)$. This implies that $p°(\lambda) > P(\lambda)$. Finally, (A6) implies that $\beta°(\lambda) = p°(\lambda)(F + R)$ (meaning that the optimal reward is $r°(\lambda) = R$). Gathering these arguments, we have $P(\lambda)(F + R) < \beta°(\lambda) < B(\lambda)$, which, by lemma 2, occurs only for $\lambda < \lambda_0$.
(ii) Assume that the optimal solution $\beta°(\lambda)$ and $p°(\lambda)$ induces $\mu = \eta = 0$. From (A3), $\phi(\beta°(\lambda), \lambda) = 0$. By lemma 1, $\beta°(\lambda) = B(\lambda)$. Using (A4), $c(p°(\lambda)) = \lambda F/(1 + \lambda)$. Hence, $p°(\lambda) = P(\lambda)$. Finally, (A5) and (A6) imply that $p°(\lambda) F \leq \beta°(\lambda) \leq p°(\lambda)(F + R)$. (This means that the optimal reward is $r°(\lambda) \in [0, R]$). Gathering these arguments, we have $P(\lambda) F \leq \beta°(\lambda) = B(\lambda) \leq P(\lambda)(F + R)$, which, by lemma 2, occurs only for $\lambda_0 \leq \lambda \leq \lambda_1$.
(iii) Assume that the optimal solution $\beta°(\lambda)$ and $p°(\lambda)$ induces $\mu > 0$. From (A3), $\phi(\beta°(\lambda), \lambda) = -\mu < 0$. By lemma 1, this implies that $\beta°(\lambda) > B(\lambda)$. Using (A4), $c(p°(\lambda)) < \lambda F/(1 + \lambda)$. This implies that $p°(\lambda) < P(\lambda)$. Finally, (A5) implies that $\beta°(\lambda) = p°(\lambda) F$. (This means that the optimal reward is $r°(\lambda) = 0$). Gathering these arguments, we have $B(\lambda) < \beta°(\lambda) < P(\lambda) F$, which, by lemma 3, occurs only for $\lambda > \lambda_1$. **QED**

A.4. Proof of proposition 3. From the proof of proposition 2, the threshold λ_1 satisfies

$$\phi(\beta, \lambda_1) = (h - \beta)\, g(\beta) - \lambda_1 (G(\beta) + \beta\, g(\beta)) = 0, \tag{A7}$$

$$(1 + \lambda_1)\, k\, c'(p) - \lambda_1 F = 0, \tag{A8}$$

([13]) Notice that $\phi(0, \lambda) = h\, g(0) > 0$ and $\phi(h, \lambda) = -\lambda (G(h) + h\, g(h)) \leq 0$, with equality only for $\lambda = 0$.
([14]) Otherwise, there would exist a sequence $\lambda_i \to \infty$ such that $B(\lambda_i) \geq k$, for some $k > 0$ and all i, and $\phi(B(\lambda_i), \lambda_i))$ would become negative for sufficiently large λ_i.
([15]) Note that, if $\beta > 0$, $\phi(\beta, \lambda)$ is (strictly) decreasing in λ. Thus, if $\lambda < \lambda'$, then $\phi(B(\lambda), \lambda') < \phi(B(\lambda), \lambda) = 0$. As $\phi(\beta, \lambda)$ is (strictly) decreasing in β, for all $\beta \geq B(\lambda)$, $\phi(\beta, \lambda') \leq \phi(B(\lambda), \lambda') < 0$. This proves that $B(\lambda') < B(\lambda)$.
([16]) Use $P(0) = 0$, $B(0) = h$, $P(\lambda) \to (c')^{-1}(F) > 0$ as $\lambda \to \infty$ and $B(\lambda) \to 0$ as $\lambda \to \infty$.

$$p\,F - \beta = 0. \tag{A9}$$

By the implicit function theorem, this defines λ_1 as a function of F, h and k, which can be characterized by differentiation. Omitting the arguments of the functions $\phi(\beta, \lambda_1)$, $g(\beta)$ and $c(p)$, for the sake of brevity, the linearized system is

$$\phi_1\, \mathrm{d}\beta = -\, \phi_2\, \mathrm{d}\lambda_1 - g\, \mathrm{d}h, \tag{A7'}$$

$$(1 + \lambda_1)\, k\, c''\, \mathrm{d}p = -\, (k\, c' - F)\, \mathrm{d}\lambda_1 + \lambda_1\, \mathrm{d}F - (1 + \lambda_1)\, c'\, \mathrm{d}k. \tag{A8'}$$

$$-\, \mathrm{d}\beta + F\, \mathrm{d}p = -\, p\, \mathrm{d}F. \tag{A9'}$$

Multiplying (A9') by $\phi_1\,(1 + \lambda_1)\, k\, c''$, subsituting (A7') and (A8'), and rearranging, we obtain

$$A\, \mathrm{d}\lambda_1 = -\, [p\,(1 + \lambda_1)\, k\, c'' + \lambda_1\, F]\, \phi_1\, \mathrm{d}F - (1 + \lambda_1)\, k\, c''\, g\, \mathrm{d}h + (1 + \lambda_1)\, c'\, F\, \phi_1\, \mathrm{d}k$$

where $A = (1 + \lambda_1)\, k\, c''\, \phi_2 - F\,(k\, c' - F)\, \phi_1 = (1 + \lambda_1)\, k\, c''\, \phi_2 + F\, k\, c'\, \phi_1/\lambda_1$ (using (A8)).

By assumption of concavity, $\phi_1(\beta, \lambda) \leq 0$ (with the strict inequality almost everywhere). Remember that the solution to the system (A7) to (A9) is such that $\beta > 0$, $p > 0$ and $\lambda_1 > 0$. Thus, $\phi_2(\beta, \lambda) = -\,(G(\beta) + \beta\ g(\beta)) < 0$ and $A < 0$. This proves that λ_1 is decreasing in F, and increasing in h and k. **QED**

Appendix B.

Sufficient condition for strict concavity of *S*(*p*, *f*, *r*) with respect to *r*.

A sufficient condition is $S_{33}(p, f, r) < 0$, for all p, f and r. As $S_3(p, f, r) = p\ \phi(p\ (f + r), \lambda)$ and $S_{33}(p, f, r) = p^2\ \phi_1(p\ (f + r), \lambda)$, this is equivalent to

$$\phi_1(\beta, \lambda) < 0, \text{ for all } \beta = p\ (f + r) \in I = [0, F + R].$$

Computing this derivative ([17]) and rearranging, we obtain

$$-\, \beta\, g'(\beta)/g(\beta) + h\, g'(\beta)/g(\beta)/(1 + \lambda) < 1 + \lambda/(1 + \lambda). \tag{B1}$$

Notice that: the LHS is smaller than $e + t\,h$, where we define $e = \sup\{-\,\beta\, g'(\beta)/g(\beta);\ \beta \in I\}$ and $t = \sup\{g'(\beta)/g(\beta);\ \beta \in I\}$; and the RHS is larger than 1. Hence, (A.1) will be true in particular if

$$e + t\,h < 1. \tag{B2}$$

According to this condition, which is clearly more stringent than necessary, only density functions $g(b)$ whose elasticity $\varepsilon(b)$ is less than 1 and whose growth rate $g'(b)/g(b)$ is bounded above, over the domain $b \in [0, F + R]$, ought to be considered. Then, either $g'(b) \leq 0$, for all b, in which case lemma 1 imposes no condition on h, or $g'(b) > 0$, for some b, in which case it requires that the harm h be sufficiently small (precisely, $h < (1 - e)/t$, with $e < 1$ and $0 < t < \infty$).

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([17]) The derivative is $\phi_1(\beta, \lambda) = -\, g(\beta) + (h - \beta)\, g'(\beta) - \lambda\,(2\, g(\beta) + \beta\, g'(\beta))$.

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