A Model of Bounded rationality

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Abstract:

This paper presents a model of individual decision with bounded rationality. The decision maker is assumed to have a limited access to information and/or a limited computation skill. The two features are analysed separately and jointly. A rough measure of the precision of the decision process is defined. It gives rise to the calculus of an economic value of the precision and, assuming that the precision of information and/or computation is costly, of an optimal precision of the decision process.

JEL classification: D80.

1. Introduction:

Simon (1955) gives the following definition of what we should mean with a model of choice with bounded rationality: “Broadly stated, the task is to replace the global rationality of economic man with a kind of rational behaviour that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist.” (Simon, 1955, p.99)

It is worth noting that, since Simon’s seminal contribution, most economists have recognized the empirical relevance of bounded rationality, but most economic theories still assume substantive rationality. Rubinstein (1998, p.4) gives two possible explanations for this state of affairs: “The evaluation that very little has been achieved makes one wonder whether it is at all possible to construct interesting models without the assumption of substantive rationality. Is there something fundamental that prevents us from constructing useful bounded rationality models, or have we been ‘brainwhashed’ by our conventional models?”

In this paper, we present a model of individual decision-making compatible with both aspects of Simon’s definition (i.e. a bounded rationality as due to a limited access to information and/or to a limited computational capacity). Following MacLeod (2002), we split up the decision-making process in two chronological steps. The ‘period of the decision-making’ is dedicated to the planning of the choice. In more precise words, at that time, ignoring the actual state of the world, the decision-maker designs a device in order to process the signals received from nature and/or computes a set of contingent plans. The next step begins when he receives a signal about the state of the world. During the ‘period of the choice’, he updates his belief and implements a choice, using his personal skills together with the tools from the first period.

The paper is organized as follows. Section 2 clarifies the hypotheses. Section 3 sets out the model, which generalizes an example presented by Marschak (1954, Sect. 3.3, Ex. II, p.202, and Sect. 5.5, p.214). With respect to Marschak, we highlight new interpretations (showing that the model is compatible with the idea of a limited computational skill), extend slightly the formalization (notably, whereas Marschak only considers given and equal partitions of the set of the states of the world, we amend this assumption in order to deal with the determination of the optimal partition) and go into greater depth into the analysis (we define a cost of the precision and deduce an optimal organization). Section 4 derives the optimal protocol.

2. Hypotheses:

Our analysis is based on Marschak (1954, Sect. 3.3, Ex. II, p.202, and Sect. 5.5, p.214). He considers a decision problem, that is, the choice of an action from a given set, in order to maximise a given objective function. He studies the value of the amount and of the precision of information. His hypotheses are [(1)]:

- the state of the world is a random variable distributed uniformly over an interval;
- the latter is partitioned in *n* equal intervals;
- the team only observes in which interval the variable falls;
- the team chooses the best action subject to that observation.

Under these assumptions, the decision schedule is bound to be a ‘step function’ (cf. Fig 1), due to the limited precision of the information. Then, Marschak defines the value of the information as the difference between the surplus of the team when $n > 1$ with its surplus when $n = 1$.

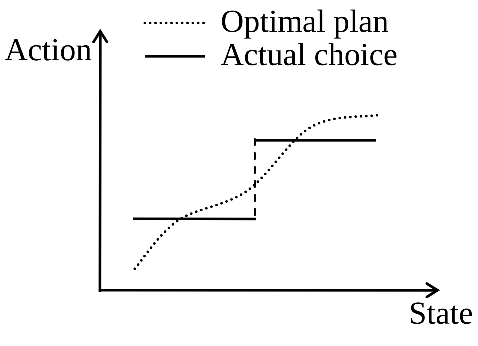

Figure 1.

([1]) Marschak (1954) analyses decisions which involve a team. But in example II, it is implicitly seen as a single entity and, since they all share the same information, coordination problems inside the team disappear.

Subject to this interpretation, Marschak's example highlights one dimension of bounded rationality (i.e. limited access to information) and neglects the other. Simon (1978, p.11) notes that this is common in the theories of teams (Marschak, 1954 and 1955; Radner, 1962) and of search (Stigler, 1961): "the bounds on the rationality of the team members are 'externalized' and represented as costs of communication".

We would like thereafter to reverse the direction of the causality. More precisely, we argue that the same result ensues whenever the optimization is assumed impossible (or imperfect, if we generalize) after the signal about the state of the world is known. For example, consider the following justification, among others. Assume first that the decision problem goes on for a finite time only (i.e. the utility of any choice becomes zero after that time). Then, following MacLeod (2002), divide the decision-making process into two periods, located with respect to the date at which the individual gets the signal about the state of the world. Finally, assume that the decision-maker has a limited computational speed, relatively to the duration of the second period, to such an extent that he cannot determine the optimal choice during the time left after he receives the signal. As a consequence, he must content himself with a choice from a set of contingent plans inherited from the first period. And, setting apart the idea of a complete smooth list of contingent plans, it ensues from these assumptions that the choice always looks like a decision schedule of the type shown in figure 1.

In the rest of the paper, we combine both interpretations, that is, limited information and limited computation. The performance of the whole decision-making process is assumed to be bounded in turn by the precision of the information, by the number of contingent choices computed during the first period, or by both simultaneously.

In the section 3, we study alternatively the three cases to obtain: 1) the optimal list of contingent actions for any given partition of the states of the world; 2) the optimal partition of the states of the world for any given list of contingent plans; 3) the optimal protocol for any given precision of the whole process of decision.

In the section 4, we go a step further to assume that the production of more precision is costly. We begin with a discussion on the benefit and on the cost of finer protocol. We then derive the optimal protocol.

3. A model using quadratic forms:

Let us now define a formal model that fits with the method described previously. It generalizes Marschak (1954, Sect. 3.3, Ex. II, p.202, and Sect. 5.5, p.214). We extend it to encompass a more general utility function and any partition (i.e. not only equal partitions) of the set of the states of the world.

Our hypotheses are:

- the decision-maker controls the value of an action x, with $x \geq 0$;
- nature chooses randomly the value of a parameter c;
- the objective is to maximise the difference $TR(x) - TC(x)$, where $TR(x) = (a - b\,x/2)\,x$ is total revenue and $TC(x) = c\,x$ is total cost;

- the parameter c is uniformly distributed between c^- and c^+;
- $a \geq c^+$, in order to rule out corner solutions.

3.1. Complete decision-making process:

The decision-making process will be said to be *complete* when two conditions are met. First, the process of the signal permits the individual to distinguish, *ex post*, any two distinct states of the world. Second, the list of contingent plans contains a distinct and optimal plan for each state of the world. Thus, a complete decision-making process implements the first best decision.

Formally, when complete planning is available, the decider measures exactly, *ex post*, the value taken by c and implements the optimal plan $x°(c)$, given by ([2]):

$$MR(x°(c)) = a - b\, x°(c) = c = MC(x°(c)), \text{ for all } c \in [c^-, c^+].$$

The net benefit attached to the state of the world c is then:

$$W(c) = TR(x°(c)) - TC(x°(c)) = (a - c)^2/2b, \text{ for all } c \in [c^-, c^+].$$

Ex ante, the expected surplus associated to the complete decision-making process is given by:

(1) $$W° = \int_{c^-}^{c^+} (a - t)^2/2b \,\mathrm{d}t/(c^+ - c^-) = ((a - c^-)^3 - (a - c^+)^3)/(6\, b\, (c^+ - c^-)).$$

3.2. Incomplete decision-making process:

The decision-making process will be said incomplete when one of the previous conditions is violated. In order to keep things tractable, we limit it, when incomplete, to satisfy some *ad hoc* restrictions. So, an incomplete decision-making process is given by [(3)]:

- a natural number: $n \geq 1$;
- a list of n contingent plans: $x_1 > x_2 > \ldots > x_n$;
- a partition of the set of the states of the world: $c_0 = c^- < c_1 < \ldots < c_{n-1} < c^+ = c_n$;
- the ability to apply, *ex post*, the contingent plan x_i in interval $[c_{i-1}, c_i[$, $i = 1, 2, \ldots, n$.

The decision-maker controls *ex ante* the three items of the process. He chooses the number n, the list of contingent plans $x_1 > x_2 > \ldots > x_n$ and the partition $c_0 = c^- < c_1 < \ldots < c_{n-1} < c^+ = c_n$. His objective is to maximize the expected surplus [(4)]:

(*) $$W(n, X, C) = \Sigma_{1 \leq i \leq n}\, (a - b\, x_i/2 - (c_i + c_{i-1})/2)\, x_i\, (c_i - c_{i-1})/(c^+ - c^-),$$

where $X = (x_1, x_2, \ldots, x_n)$ and $C = (c_0, c_1, \ldots, c_n)$.

([2]) Of course, $MR(x)$ and $MC(x)$ respectively stand for marginal revenue and marginal cost.

([3]) The condition that the list of contingent plans is strictly increasing can be justified further. First of all, the problem is not well defined if the n contingent plans are not distinct. Moreover, it is immediate to demonstrate that $x_i < x_j$, with $i < j$, is always inefficient: the smaller the marginal cost is, the bigger the action is.

([4]) To derive this expression, let us first note that, *ex post*, the surplus is: $TR(x_i) - TC(x_i) = (a - b\, x_i/2 - c)\, x_i$, for all c in $[c_{i-1}, c_i[$ ($i = 1, 2, \ldots, n$). The expected surplus is thus given by: $W(n, X, C) = \Sigma_{1 \leq i \leq n} \int_{c_{i-1}}^{c_i} (a - b\, x_i/2 - t)\, x_i \,\mathrm{d}t$. Integration yields the result.

The analysis of the choice of n is undertaken below. For the moment, it is assumed given. Subject to this restriction, let us study the remaining decisions.

o Choosing *X* alone (with *C* given):

To begin with, let us consider the case where the process aimed at collecting the information is given. Precisely, let us suppose that it classes the states of the world with respect to the partition $c_0 = c^- < c_1 < \dots < c_{n-1} < c^+ = c_n$. The decision-maker has to fit the list of contingent plans $x_1 > x_2 > \dots > x_n$ with this partition.

Given any $C = (c_0, c_1, \dots, c_n)$, the list of contingent plans $X(C) = (x_1(C, n), x_2(C, n), \dots, x_n(C, n))$ maximizes the individual's objective function if and only if it satisfies the conditions:

$$\partial W(n, X, C)/\partial x_i = (a - b\, x_i(C, n) - (c_i + c_{i-1})/2)\, (c_i - c_{i-1})/(c^+ - c^-) = 0, \text{ for all } i = 1, 2, \dots, n.$$

As $c_i - c_{i-1} > 0$, we obtain:

(2) $$x_i(C, n) = (a - (c_i + c_{i-1})/2)/b, \text{ for all } i = 1, 2, \dots, n.$$

This result has a straightforward interpretation. The optimal contingent plan $x_i(C, n)$, designed for the range $[c_{i-1}, c_i[$, is such that the benefit of a marginal unit equals its expected cost of production, which is equal to $(c_i + c_{i-1})/2$ (given that $c \in [c_{i-1}, c_i[$).

Substituting (2) for x_i ($i = 1, 2, \dots, n$) in the objective function (*), we get:

$$W(n, X(C, n), C) = \Sigma_{1\le i\le n}\, (a - (c_i + c_{i-1})/2)^2\, (c_i - c_{i-1})/(2\, b\, (c^+ - c^-)),$$

which gives the maximum surplus compatible with the given partition.

o Choosing *C* alone (*X* given):

We can also consider the case where the list of contingent plans $x_1 < x_2 < \dots < x_n$ is fixed (with $(a - c^+)/b \le x_i \le (a - c^-)/b$ to rule out corner solutions). Subject to this constraint, the decision-maker is left with the task of designing the informational instrument that best fits with it. We assume that he can choose any partition $c_0 = c^- < c_1 < \dots < c_{n-1} < c^+ = c_n$ he wants.

To solve this problem, notice that the term c_k ($k = 1, 2, \dots, n - 1$) appears in the objective (*):

- at rank $i = k$: $(a - b\, x_k/2 - (c_k + c_{k-1})/2)\, x_k\, (c_k - c_{k-1})$
- at rank $i = k + 1$: $(a - b\, x_{k+1}/2 - (c_{k+1} + c_k)/2)\, x_{k+1}\, (c_{k+1} - c_k)$

Partial differentiation with respect to c_i thus yields (after simplification):

$$\partial W(n, X, C)/\partial c_i = -\,(a - b\, (x_{i+1} + x_i)/2 - c_i)\, (x_{i+1} - x_i)/(c^+ - c^-), \text{ for all } i = 1, 2, \dots, n - 1.$$

Given any $X = (x_1, x_2, \dots, x_n)$, the partition $C(X, n) = (c_0(X, n), c_1(X, n), \dots, c_n(X, n))$ yields the maximum expected surplus if and only if $\partial W(n, X, C)/\partial c_i$, evaluated at $c_i(X, n)$, is zero (for an interior solution). We thus have:

$$c_0(X, n) = c^-,$$

(3) $$c_i(X, n) = a - b\,(x_{i+1} + x_i)/2, \text{ for all } i = 1, 2, \ldots, n-1,$$

$$c_n(X, n) = c^+.$$

If we substitute this result for c_i ($i = 0, 1, \ldots, n$) in (*), we obtain:

$$W(n, X, C(X, n)) = \{((a - b\,x_1/2 - c^-)^2 - (b\,x_2/2)^2)\,x_1 + (b^2/4)\,\Sigma_{2\leq i\leq n-1}\,x_i\,(x_{i+1}^2 - x_{i-1}^2) + ((a - b\,x_n/2 - c^-)^2 - (b\,x_{n-1}/2)^2)\,x_n\}/2(c^+ - c^-),$$

which gives the maximum surplus compatible with the given list of contingent plans.

o **Choosing both *X* and *C*:**

Now, let us suppose that the decision-maker is in a position to choose both the partition and the list of contingent plans to maximize his objective.

Then, the list of contingent plans $X(n) = (x_1(n), x_2(n), \ldots, x_n(n))$ and the partition $C(n) = (c_0(n), c_1(n), \ldots, c_n(n))$ maximizes the objective function if it satisfies both conditions (2) and (3). Substituting (2) into (3) yields the following system of linear equations:

$$c_0(n) = c^-,$$

$$c_{i+1}(n) - 2\,c_i(n) + c_{i-1}(n) = 0, \text{ for all } i = 1, 2, \ldots, n-1,$$

$$c_n(n) = c^+.$$

It has a unique solution given by $c_i(n) = c^- + (c^+ - c^-)\,i/n$, for $i = 0, 1, \ldots, n$. We conclude that the optimal decision-making scheme, for any n, is unique and is determined by:

(4.a) $$x_i(n) = (a - c^- - (c^+ - c^-)\,(i - 1/2)/n)/b, \text{ for all } i = 1, 2, \ldots, n.$$

(4.b) $$c_i(n) = c^- + (c^+ - c^-)\,i/n, \text{ for all } i = 0, 1, \ldots, n.$$

The figure 2 illustrates this result. We assume that $c^- = 0$, $a = b = c^+ = 1$ and $n = 2$. The optimal list of contingent plans is: $X(2) = (3/4, 1/4)$.The optimal partition is: $C(2) = (0, 1/2, 1)$.

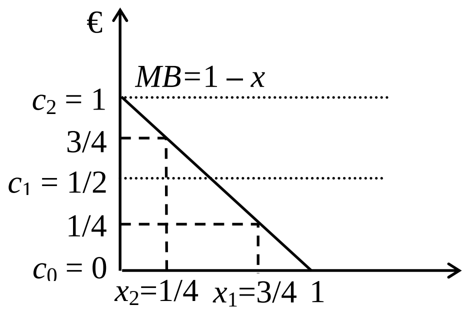

Figure 2.

Finally, introducing this strategy into (*), we obtain the maximum expected surplus [5]:

(5) $W(n) = W(n, X(n), C(n)) = W° – (c^+ – c^–)^2/(24\, b\, n^2)$,

with $W° = ((a – c^–)^3 – (a – c^+)^3)/(6\, b\, (c^+ – c^–))$ given by (1).

4. Optimal precision of the decision-making process:

We now complete our explanation of the choice of the decision-making process. We are concerned with the choice of *n*. In a first step, we derive the effects of a variation of *n* on the expected surplus. In a second, we argue that the cost of the planning procedure ought to be a function of *n*. Finally, we gather these elements to find the best process.

4.1. Benefit of precision:

Let us recall that a unit in *n* means that the decision-maker improves the processing of information and the planning, to such an extent that he is able to apply *ex post* one more contingent plan in a given range of the states of the world. So, the bigger *n* is, the finer is the process.

To begin with, using (5), we easily confirm some intuitive (but nevertheless interesting) results. The difference between the expected surplus with a complete decision-making process and with an incomplete decision-making process is:

$$W° – W(n) = (c^+ – c^–)^2/(24\, b\, n^2) = (V/2b)\,(1/n^2) > 0,$$

with $V = (c^+ – c^–)^2/12$ the variance of *c*. The difference is positive, increasing in *V* and decreasing in *b* and *n*. At the limit, when *n* tends to $+\infty$, W_n tends to $W°$.

A more useful case is the benefit of a marginal contingent plan. We can employ (5) to define it:

$$b(n) = W_{n+1}^* – W_n^* = (V/2b)\,((2\,n + 1)/(n\,(n + 1))^2).$$

It is immediate that $b(n)$ is positive, increasing in *V* and decreasing in *b* and *n*. It tends to 0 when *n* tends to $+\infty$. Thus, the finer the decision-making process is, the less productive is the marginal contingent plan.

3.2. Costs of precision:

Without loss of generality, we postulate that the cost of planning is a function of *n*, *X* and *C*. Given the lack of empirical evidences, we ignore what this relation looks like in reality. Nevertheless, we argue that it should approximately be an increasing function of *n*.

Indeed, to increase *n*, the decider must: compute new contingent plans; invest in a finer information technology since the thread $\{\min\{|c_i – c_j|;\ i \neq j\}$ decreases; and implement more

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[5] To show this, we use (*), (4a) and (4b) to get: $W(n) = (c^+ – c^–)^2/(2bn^3)\ \Sigma_{1\le i\le n}\,(n\,(a – c^–)/(c^+ – c^–) + 1/2 – i)^2$. Then, recall that $\Sigma_{1\le i\le n}\, i = n\,(n + 1)/2$ and $\Sigma_{1\le i\le n}\, i^2 = n\,(n + 1)\,(2n + 1)/6$, introduce these results into the sum and simplify to get:

$$\begin{aligned} W(n) &= (c^+ – c^–)^2/(2bn^3)\ \{[((a – c^–)/(c^+ – c^–))^2 – (a – c^–)/(c^+ – c^–) + 1/3]\ n^3 – n/12\} \\ &= 1/6b\ \{3\,(a – c^–)^2 – 3\,(a – c^–)\,(c^+ – c^–) + (c^+ – c^–)^2 – (c^+ – c^–)^2/4n^2\} \\ &= 1/6b\ \{((a – c^–)^3 – (a – c^+)^3)/(c^+ – c^–) – (c^+ – c^–)^2/4n^2\} \end{aligned}$$

seemingly contingent plans since $\min\{|x_i - x_j|;\ i \neq j\}$ decreases, enhancing the risks of errors. All these effects give rise to an increase of the cost of planning.

These elements lead us to consider thereafter that the cost of a marginal contingent plan $c(n)$ is positive and is not decreasing in n.

3.3. Optimal decision-making process:

The optimal decision-making process results from a standard marginal argument. As long as $b(n) > c(n)$ (resp. $b(n) < c(n)$), the individual improves (decreases) his objective function if he provides one more contingent plan. The optimal process is thus given by the unique integer n^* so that (the uniqueness follows from the fact that $b(n)$ is strictly decreasing in n and that $c(n)$ is not decreasing in n):

$$c(n^*) < b(n^*) \text{ and } b(n^* + 1) < c(n^* + 1).$$

Using the expression $b(n) = (V/2b)\ ((2\ n + 1)/(n\ (n + 1))^2)$, it is immediate that others things being equal, the optimal precision of the decision-making process is increasing in V and decreasing in b. Moreover, it stays constant if V and b increase proportionally (i.e. V/b is constant).

Bibliography:

MacLeod, W.B., 2002, Planning Costs and the Theory of Learning by Doing, University of Southern California Research Paper No. C03-16.

Marschak, J., 1954, Towards an Economic Theory of Organization and Information, in: Thrall and al., eds., Decision Processes, John Wiley, 1954.

Radner, R., 1962, Team Decision Problems, Annals of Mathematical Statistics, 33(3), 857-881.

Rubinstein, A., 1998, Modelling Bounded Rationality, The MIT Press.

Simon, H.A., 1955, A Behavioral model of rational choice, Quaterly Journal of Economics, 69, 99-118.

Simon, H.A., 1978, Rationality as Process and as Product of Thought, American Economic Review, 68(2), 1-16.

Simon, H.A., 1979, Rational Decision Making in Business Organizations, American Economic Review, 69(4), 493-513.

Simon, H.A., 1994, Bounded rationality, in: R.H.I. Palgrave and al., The New Palgrave: a Dictionary of Economics, vol. 1, Macmillan, p. 266-268.