Recycling Tax Revenue to Support Pollution Abatement under Asymmetric Information

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This paper proposes to characterize the regulation of water pollution in France. Our main concern is to test the efficiency of an environmental policy that combines the use of a tax and a contractual complement, which matches the investments undertaken by the polluters and public aids. Our modelisation takes into account simultaneously the firms' informational advantages about their abatement costs and the need for the regulator to balance his budget. Our results show that: this policy can be costly if a too small tax rate is chosen (compared to the first best policy); the first best policy can be implemented if the tax rate is chosen above a critical level; the latter is smaller than the Pigouvian tax. For each situation, a precise description of the contract is provided (allocation of abatement efforts, relation between abatement and aid). This gives us the opportunity to discuss the place of Water Agencies from an institutional point of view.

1. INTRODUCTION

In France, the regulation of water pollution has been delegated to six regional Water Agencies since 1964. They are public establishments. They define a five year program which fixes the objectives to be reached and the amount of investment aid necessary to achieve them. To this respect, they are endowed with a financial autonomy. As a result, the tax rate is calculated so as to cover the commitments made by the Agency, taking into account the recovery of loans and advances which have been granted investors.

Each Water Agency is endowed with the right 1) to tax the polluters and 2) to use the tax revenue to subsidy abatement projects within a contractual scheme. With respect to the first remit, practical difficulties lead the agency to behave in a way that is far from the text-book prescription. On the one hand, the tax base is indirectly correlated to the emission since it is computed with respect to the average day of the most productive month. This imperfection follows from the impossibility to monitor the emissions at a reasonable cost. On the other hand, the tax rate is fixed without any reference to the pigouvian level. The main reason is the political acceptability of the regulatory system.

The second remit is a way to improve the efficiency of the regulation, subject to the above constraint (i.e. a low tax rate). The mechanism operates through a communication round and an exchange relation between the firm and the Water Agency, during which the abatement project is proposed, evaluated, monitored and financed. The aids reduce the financial burden incurred by the firm and can take several forms: a subsidy, a loan or an advance.

This paper proposes a rigourous treatment of this regulatory scheme, called *Affectation Principle* thereafter, which definition follows. Firstly, the affectation principle is a sequential and mixed instrument: in a first (fiscal) step, the emission tax is chosen; in the second (contractual) step, the tax revenue is distributed in exchange for an abatement objective. Secondly (and this justifies the above two steps time sequence), the Water Agency balances the budget: the subsidies are self-financed by the industry. These points are developpped in section 2 and 4.

It is important to recognize that the second step of the Water Agency's regulation works efficiently if the firm communicates his information correctly to the principal. Except if the firm is intrinsically honest, this should not be the case, unless the Water Agency respects few careful rules, specifically designed to induce the correct behavior (see section 3). Otherwise, the regulator could suffer the opportunism of the firm and thus fail to improve the social welfare.

This observation is not new and means that this problem must be treated within the principal-agent theorical framework (unless we assume a civic behavior from the firm). This tool was first developped in the seminal article of Baron and Myerson [1], to deal with the regulation of a monopoly privately informed of his production cost. The first application to the regulation of pollution are due to Baron [2] and Spulber [8] [1]. Recent contributions to this field are those of Thomas [9], Lewis [7], and Jebjerg and Lando [5].

The affectation principle has not yet received much attention in the literature. One exception is Thomas [9], but his analysis only deals with the first part of the definition (the combined use of a tax and a contract) and sets aside the second part. With respect to the results of the regulation, however, the budget constraint is an important feature of the affectation principle, as we show in section 5. When introduced in the analysis, the budget constraint produces an endogeneous source of an allocation inefficiency of the regulation [2]. The best policy must concentrate the abatement effort on the most efficient types, in order to limit informational rents and thus the transfers to be paid to the firm. The consequence of a budget constraint is studied in Spulber [8] and only skimmed over in Lewis [7], but in both cases, the modelisation does not fit the

[1] This list sets aside few important *older* contributions (for example, Kwerel [4], Dasgupta and al. [3]). The reason of this omission is methodological, since the latter use a dominant strategy equilibrium concept (Clarke-Groves mechanisms) instead of a Nash-Bayes equilibrium concept.

[2] In Thomas [9], the allocative inefficiency is exogeneous, since it results from a distributive arbitrage between consumers and firms. This idea is developed in Baron [2].

affectation principle [3].

2. GENERAL BACKROUND

In this paper, the intervention of the Water Agency is seen as a three steps process (See Fig. 1). Each step represents a decision node. The chronology matters.

Actor:	principal	–>	principal	–>	agent
Decision:	effluent fee		menu of contracts		contract

Legend: Principal = Water Agency
Agent = Firm

FIG.1. Actors and chronology of decisions

In a first step, the principal chooses an effluent fee. An important element here is the bad popularity of this instrument. Given this problem of acceptability, it is legitimate to assume that the principal can not enforce any tax level. Thereafter, we assume that it must be kept below a given limit (smaller than the Pigouvian tax). An immediate consequence of this political constraint is the inefficiency of the regulation (at least if it is not completed with the contractual mechanism).

In a second step, the principal designs and proposes to the agent a contractual mechanism. It should be seen as a menu of contracts, specifying simultaneously a more stringent abatement objective and a compensatory transfer. This instrument is added on to the effluent fee. Of course, the adjonction of this second instrument can only improve the global efficiency of the regulation (again, the tax being given and assumed too small), since we can not expect the regulator to use it otherwise.

Nevertheless, and this is the central feature of our analysis, this mechanism suffers from an institutional limit. The principal must refrain himself in order to stay within the budget, which is given by the tax revenue he collects. Therefore, he may not be able to finance every project he would like. In that respect, if the budget is to be balanced (as it is asssumed), the problem of the choice of the effluent fee reappears in another form, as it limits the regulator's actions in the second step.

In a third step, the agent can accept or refuse to sign a contract. If he accepts, he must conform to it. If he refuses, he decides unilaterally his abatement. But, in both cases, the effluent fee is paid.

Let us now introduce the model. Let z denote the agent's maximum pollution (when he does not intend to reduce it). Pollution has a detrimental effect on welfare, figured by the cost of damage $D(z)$. We assume that the cost of damage is linear, with

[3] In Spulber [8], the budget is said to be balanced if the consumers can compensate the firms' informational rents (that is, if the net surplus is positive).

$D'(z) = \tau^*$ [4]. This assumption sounds legitimate if the agent's contribution to the global pollution is small with respect to the others. As Jebjerg and Lando [5] note, it allows us "to interpret [the model] as covering the regulation of an industry with many firms, since the pollution of one firm does not affect the optimal tax scheme for another firm under linearity". Using this information, we derive the benefit of an abatement of an amount q, $S(q) = \tau^* q$.

The agent can reduce his effluent. Let $\theta c(q)$ be the cost of abatement incurred where q is the avoided quantity. This formulation is very general, as it implicitly includes all abatement methods one can imagine: input substitutions, end-of-pipe processes, development of clean technologies... The abatement cost is constrained to be increasing and convex in the quantity. We also impose the following properties for practical and/or technical reasons: $c(0) = 0$, $c'(0) = 0$ and $c'(z) = \infty$.

An important assumption is that θ is a private information, that is, it is known to the agent only. The principal benefits only from a partial information. We assume that he knows or believes that θ is distributed on $\Theta = \left[\underline{\theta}, \overline{\theta}\right]$, with a cumulative distribution $F(\theta)$ and a density $f(\theta) > 0$.

Formally, the principal's instruments are an effluent fee τ and a subsidy A. These instruments influence the agent's decision through their effect on his profit:

$$\pi = -\tau(z - q) - \theta c(q) + A$$

Note that this expression implicitely suggests that the effluent fee is paid *ex post*, as it is equal to $\tau(z - q)$ and necessitates that the effective abatement is known. This is only an interpretationof the model. For example, the effluent fee could be paid *ex ante*. In such a case, as the actual abatement is unknown, the firm pays τz; *ex post* the result below applies if we define $A' = A + \tau q$, that is, if the excess tax is implicitly reimbursed. Of course, this makes no difference with respect to our results. But it shows that the model can receive several interpretations.

3. AGENT'S DECISIONS

This section deals with the third decision node in the above figure. It is dedicated to the choice of the agent. Two situations are considered.

In the first one, the agent refuses any contract and prefers to act alone. He chooses his abatement effort taking into account the effluent fee only.

In the second one, the agent participates in the mechanism proposed by the principal. He chooses a contract taking into account the effluent fee (which is due), and the terms of the contracts, that is, the abatement and the corresponding subsidy.

3.1.The statu quo

[4] The marginal cost of damage could also be interpreted as the shadow price of an exogeneously given environmental objective. In particular, this interpretation seems appropriate for the French regulation of water pollution, since it is expected to comply with European norms.

Under the statu quo, the agent solves the program $\max_q\left(\tau\left(q-z\right)-\theta c\left(q\right)\right)$. Let us define $R\left(\theta,\tau\right)=\max_q\left(\tau\left(q-z\right)-\theta c\left(q\right)\right)$ and $\rho\left(\theta,\tau\right)=\arg\max_q\left(\tau\left(q-z\right)-\theta c\left(q\right)\right)$ respectively the statu quo (reservation) utility and abatement of the agent.

It is immediate to show that 1) the reservation utility is decreasing with the type and the fee (i.e. $\partial R\left(\theta,\tau\right)/\partial\theta<0$ and $\partial R\left(\theta,\tau\right)/\partial\tau<0$) and 2) the statu quo abatement is decreasing with the type and increasing with the fee (i.e. $\partial\rho\left(\theta,\tau\right)/\partial\theta<0$ and $\partial\rho\left(\theta,\tau\right)/\partial\tau>0$).

3.2.The choice of a contract

A contract is the combination of an abatement effort and a subsidy. When the agent signs a contract, he is asked to satisfy the abatement objective ; the return of his effort is the subsidy. On the one hand, the principal designs a list of contracts to meet his goals (pollution, costs, benefits...). On the other hand, the agent picks up a contract taken from the menu by announcing the correct message in order to maximise his utility.

The revelation principle (see, for example, Baron and Myerson [1]) simplifies the problem. Since any feasible allocation can be obtained as the result of a direct mechanism, a mechanism can be written without any loss of generality:

$$C=\left\{(q(\hat{\theta}),A(\hat{\theta})):\hat{\theta}\in\Theta\right\}$$

It gives a list of targeted contracts. The agent reserves a contract by announcing a type $\hat{\theta}\in\Theta$. The contract specifies an abatement objective q and a transfer A. The mechanism is feasible: it induces the agent to participate and to announce his true type, regardless his type.

Let us note:

$$\pi(\hat{\theta},\theta)=-\tau(z-q(\hat{\theta}))-\theta c(q(\hat{\theta}))+A(\hat{\theta}),$$

the profit of the agent when he announces $\hat{\theta}$ and his type is θ.

A decision q is implementable if the principal can design a transfer A that satisfies the incentive compatibility constraint:

$$\pi(\theta)\equiv\pi(\theta,\theta)\geq\pi(\hat{\theta},\theta)\text{ for all }\hat{\theta},\theta\in\Theta^2$$

This condition can be written in a more convenient and useful form:

Implementability (I): q is non increasing with θ,
Incentive compatibility (IC): $\dot{\pi}\left(\theta\right)=-c\left(q\left(\theta\right)\right)$.

Proof. (IC) is a first order (or local) condition for optimality. To establish it, note that:

$$\pi(\theta)=\max_{\hat{\theta}\in\Theta}\ \pi(\hat{\theta},\theta)$$
$$\Rightarrow d\pi(\theta)/d\theta=\partial\pi(\theta,\theta)/\partial\theta=-c\left(q\left(\theta\right)\right)$$

(I) is a second order (or global) condition for optimality. To establish it, we use the definition of the incentive compatibility constraint and the fact that $c(q)$ is increasing in q.

$$\begin{aligned} &\pi(\theta,\theta) \geq \pi(\hat{\theta},\theta) \text{ for all } \hat{\theta},\theta \in \Theta^2 \\ &\Rightarrow \pi(\theta,\theta) - \pi(\theta,\hat{\theta}) \geq \pi(\hat{\theta},\theta) - \pi(\hat{\theta},\hat{\theta}) \\ &\Rightarrow (\hat{\theta}-\theta)c(q(\theta)) \geq (\hat{\theta}-\theta)c(q(\hat{\theta})) \end{aligned}$$

We now introduce the fact that $c(q)$ is increasing in q:

$$\left\{ \begin{array}{c} (\hat{\theta}-\theta)c(q(\theta)) \geq (\hat{\theta}-\theta)c(q(\hat{\theta})) \\ c(q) \text{ is increasing in } q \end{array} \right.$$

$$\Rightarrow \text{if } \hat{\theta} > \theta, \text{ then } c(q(\theta)) \geq c(q(\hat{\theta})) \text{ and } q(\theta) \geq q(\hat{\theta})$$

This completes the proof.

The constraints above characterize an implementable mechanism. They are not sufficient for feasibility. Since the agent can refuse the contract, the principal must induce him to participate. This implies that the mechanism must also satisfy a participation constraint:

$$\text{Participation (P):} \quad \pi(\theta) \geq R(\theta,\tau) \text{ for all } \theta \in \Theta$$

If the contraint is satisfied, it is rational to participate.

4. THE AFFECTATION PRINCIPE

We call affectation principle the power given to the principal to fix, collect and recycle the tax revenue in order to support abatement projects. A few explanations are necessary.

Let $W(\theta) = \tau^* q - \theta c(q)$ be the ex post social surplus. Because θ is a private information, the principal's objective is to maximise the expected social surplus $E_\theta[W(\theta)] = E_\theta[\tau^* q - \theta c(q)]$.

Let us first emphasize the well-known qualities of the Pigouvian tax. With no constraint on the choice of the tax, the traditional recommendation of Pigou applies. Despite the informational asymmetry, the principal implements the ex post (full information) social optimum by setting an emission tax equal to the constant marginal benefit of abatement [5]. This is because the principal's and agent's programs are then identical. The ex post efficient abatement is defined by $q^{**}(\theta) = \arg\max_q (\tau^* q - \theta c(q))$. The

[5] *Note that this result is not fully general. When the marginal benefit of abatement is decreasing, the regulator can not implement the ex post optimal abatement, since he fails to evaluate ex ante the marginal benefit of abatement. This problem is addressed for example in Weitzman [10].*

agent implements it when he faces the real cost of pollution τ^* (note that $\rho(\theta, \tau^*) = q^{**}(\theta)$).

In many situations, the principal can not or does not want to use the pigouvian tax, for political reasons mainly. In this context, the affectation principle provides the principal with a complementary instrument. The presumption is that it will help to improve the social welfare, despite the political constraint. This is the question we arise thereafter.

The following analysis argues that this approach suffers a limit because the principal is limited by his budget. At the firm level, this requirement can be written:

$$\text{Budget balance (BB): } E_\theta[\tau(z-q)] = E_\theta[A]$$

In other words, the principal can not distribute on average more subsidy than the tax revenue he collects. Of course, ex post, this constraint is not sufficient to balance the budget. Nevertheless, as the contract is applied to a lot of firms, the equilibrium will be achieved, due to the law of large numbers.

The consequence of the budget balance constraint is the appearance of an endogeneous shadow price of the public fund. So, even without any fiscal distorsion (a standard assumption in most models of regulation in the literature of mechanism design ; see for example Laffont and Tirole [6]), the system incurs an endogeneous inefficiency. Once this point is understood, there remains a classical problem of rents/efficiency arbitrage.

Note: the affectation principe is sometimes critized because it does not satisfy the Polluter-pays principe. This opinion is not correct, as far as this principe means that the polluters should bear the entire cost of depollution (not the environmental cost). In fact, note that (BB) implies that:

$$E_\theta[\pi] = -E_\theta[\theta c(q)]$$

5. MECHANISM DESIGN

Let us assume that the principal has already chosen the effluent fee at the first decision node. At the second decision node, his objective is summarized in the program:

$$\max_C \int_{\underline{\theta}}^{\overline{\theta}} [\tau^* q - \theta c(q)] \, dF(\theta) \tag{1}$$

subject to the constraints (I), (IC), (IR), (BB) and (IA): $q \geq \rho$ for all θ.

The last constraint, called (IA) for increased abatement, states that the principal can not propose the agent to decrease his abatement with respect to the statu quo situation. This assumption is made both for practical and technical reasons.

PROPOSITION 1. Under the (IA) constraint, the participation constraint (IR) is binding only at $\overline{\theta}$ if (IC) is satisfied.

COROLLARY 1. (IR) reduces to the boundary constraint $\pi\left(\overline{\theta}\right) \geq R\left(\overline{\theta}, \tau\right)$.

Proof. Let $\Delta(\theta) = \pi(\theta) - R(\theta, \tau)$ denote the difference between the profit of type θ inside and outside the contract. Suppose $\Delta\left(\overline{\theta}\right) \geq 0$. Using (IC) and the definition of ρ, note that $\dot{\Delta}(\theta) = c(\rho) - c(q)$. We conclude that $\dot{\Delta}(\theta) \leq 0$ if $q \geq \rho$. This completes the proof.

We use the optimal control theory to solve this program. The effort q is the control. The state variables are the agent's profit π and the principal's budget B. Note that we use the following expression of the budget constraint:

$$\dot{B} = (\tau(z-q) - A) f(\theta) = -(\pi + \theta c(q)) f(\theta),$$
$$B(\underline{\theta}) = 0 \text{ and } B\left(\overline{\theta}\right) \geq 0$$

Defining λ and μ respectively the two co-state Pontryagin multipliers associated with the states π and B, and η the Lagrangian multiplier associated with (IA) [6], we write the Hamiltonian:

$$H = [\tau^* q - (1+\mu)\theta c(q) - \mu\pi + \eta(q-\rho)] f(\theta) - \lambda c(q) \tag{2}$$

Let q^* denote the optimal solution. The conditions for optimality are:

$$\partial H/\partial q = [\tau^* - (1+\mu)\theta c'(q^*) + \eta] f(\theta) - \lambda c'(q) = 0 \tag{3}$$

$$\text{with } \eta \geq 0,\ q^* \geq \rho \text{ and } \eta(q^* - \rho) = 0 \tag{4}$$

$$-\partial H/\partial B = \dot{\mu} = 0 \tag{5}$$

$$-\partial H/\partial \pi = \dot{\lambda} = \mu f \tag{6}$$

$$\lambda\left(\overline{\theta}\right) \geq 0,\ \pi\left(\overline{\theta}\right) \geq R\left(\overline{\theta}\right) \text{ and } \lambda\left(\overline{\theta}\right)\left(\pi\left(\overline{\theta}\right) - R\left(\overline{\theta}\right)\right) = 0 \tag{7}$$

$$\mu\left(\overline{\theta}\right) \geq 0,\ B\left(\overline{\theta}\right) \geq 0 \text{ and } \mu\left(\overline{\theta}\right) B\left(\overline{\theta}\right) = 0 \tag{8}$$

The condition (5) implies that the co-state $\mu(\theta)$ is a constant, say $\bar{\mu}$ thereafter. In (6), we deduce that $\lambda(\theta) = \bar{\mu}F(\theta)$, where we use the fact that $\lambda(\underline{\theta}) = 0$ since $\pi(\underline{\theta})$ is unconstrained. Finally, gathering these results in (3), we get:

$$\tau^* - (1 + \bar{\mu}(1 + \Omega(\theta)))\theta c'(q^*) + \eta = 0 \tag{9}$$

where $\Omega(\theta) \equiv F(\theta) / (\theta f(\theta))$. Implicitely, this condition defines the optimal abatement level as a function of the type and the social cost of the budget constraint. We state this result in the proposition 2 (closed to Proposition 1 in Thomas [9]).

[6] *Note that (IA) is rewritten* $(q-\rho) f(\theta) \geq 0$ *in order to simplify the notation.*

PROPOSITION 2. Let $q^*(\theta, \bar{\mu})$ be the optimal abatement, solution to the principal's program. Assume that $\Omega(\theta)$ is non decreasing with θ.
1) There exists a critical value $\tilde{\theta} \in \Theta$, given by:

$$\tilde{\theta} = \begin{cases} \underline{\theta} \text{ if } \tau^*/\tau \leq 1 + \bar{\mu} \\ \overline{\theta} \text{ if } \tau^*/\tau \geq 1 + \bar{\mu}\left(1 + \Omega\left(\overline{\theta}\right)\right) \\ \text{solution to } \Omega(\theta) = (\tau^*/\tau - 1)/\bar{\mu} - 1 \text{ otherwise} \end{cases}$$

such that:

(a) for $\theta \leq \tilde{\theta}$, the firm is regulated (in the sense that $q^* > \rho$) and the optimal abatement $q^*(\theta, \bar{\mu})$ is the root of:

$$(1 + \bar{\mu}(1 + \Omega(\theta)))\,\theta c'(q) = \tau^*$$

(b) for $\theta > \tilde{\theta}$, the firm is not regulated and the abatement is limited by (IA), that is:

$$q^*(\theta, \bar{\mu}) = \rho(\theta, \tau)$$

2) The optimal abatement is both decreasing in θ (strictly) and $\bar{\mu}$ (weakly).

Proof. See the Appendix.

The result of Proposition 2 can also be written:

$$\theta c'(q^*) = \max\left(\tau^*/\left(1 + \bar{\mu}(1 + \Omega(\theta))\right), \tau\right)$$

Proof. This is a direct consequence of Proposition 2. For $\theta \leq \tilde{\theta}$, we have $\theta c'(q^*) = \tau^*/\left(1 + \bar{\mu}(1 + \Omega(\theta))\right) \geq \tau$ (equality at $\tilde{\theta}$). For $\theta > \tilde{\theta}$, q^* is given by $\rho(\theta, \tau)$, and we have $\theta c'(q^*) = \tau > \tau^*/\left(1 + \bar{\mu}(1 + \Omega(\theta))\right)$.

This formulation enlights the nature of the regulation. It shows that the smaller the efficiency parameter θ (that is, the easier it is to abate), the higher the marginal cost of abatement incurred by the firm (see Fig. 2).

FIG.2. Nature of the regulation

This feature of the regulatory scheme is a direct consequence of the budget constraint. As a starting point, note that when this constraint is not binding, all types incur the same marginal cost of abatement, set at the marginal benefit of depollution ($\bar{\mu} = 0$ implies $\theta c' (q^*) = \tau^*$, for all θ). Note also that in such a case, the optimal solution of the principal's program is the ex post efficient abatement, that is, $q^* (\theta, 0) = q^{**} (\theta)$.

These results cease to be true when the budget constraint is binding. In this case, it is optimal to concentrate the abatement effort on the most efficient types. One can check this assertion in noting that $\partial \theta c' (q^*) / \partial \theta$ is either negative or null. This allocation 1) minimizes the informational rents to be distributed in order to allocate the budget in a way that 2) stays as closed as possible to the ex post optimal allocation $q^{**} (\theta)$.

6. ARBITRAGE BETWEEN EFFICIENCY AND DISTRIBUTION

Up to here, we ignored the first step of the decision problem, that is, the choice of the fee. We now introduce this problem and try to show that it should be interpreted as an arbitrage between optimality and political acceptability.

PROPOSITION 3. There exists a critical value of the fee, say $\tilde{\tau}$, with $0 < \tilde{\tau} < \tau^*$, such that if $\tau \geq \tilde{\tau}$ (resp. $\tau < \tilde{\tau}$), the ex post optimal allocation q^{**} is (resp. is not) solution to the principal's program (with a budget contraint).

Proof. See the Appendix.

This proposition clarifies the link between the choice of the fee and the global performance of the regulation.

If the tax is too small, the principal is too "poor" to implement the first best allocation. It is interesting to note that the smaller the effluent fee, the larger the gap between the actual abatement (solution to (1)) and the ex post optimal allocation.

Proof. The optimal solution of (1), noted q^*, solves the following system of equations:

$$\begin{cases} \tau^* - (1+\bar{\mu}(1+\Omega(\theta)))\,\theta c'(q) + \eta = 0 \\ \text{with } \eta \geq 0,\ q \geq \rho \text{ and } \eta(q-\rho) = 0 \\ B\left(\overline{\theta}\right) = -R\left(\overline{\theta},\tau\right) - \int_{\underline{\theta}}^{\overline{\theta}} (1+\Omega(\theta))\,\theta c(q)\, f(\theta)\, d\theta \\ \text{with } \bar{\mu} \geq 0,\ B\left(\overline{\theta}\right) \geq 0 \text{ and } \bar{\mu} B\left(\overline{\theta}\right) = 0 \end{cases}$$

We proved in Proposition 2 that using the first equation, we can define q^* as a function of θ and $\bar{\mu}$, with $\partial q^*/\partial\bar{\mu} \leq 0$.

Into the second equation, this result implies that one can express $\bar{\mu}$ as a function of τ:

- if the budget constraint is binding, using the implicit function theorem, we get:

$$\partial\bar{\mu}(\tau)/\partial\tau = -\partial B\left(\overline{\theta}\right)/\partial\tau \big/ \partial B\left(\overline{\theta}\right)/\partial\bar{\mu} \leq 0$$

where:

$$\begin{aligned} \partial B\left(\overline{\theta}\right)/\partial\tau &= -\partial R\left(\overline{\theta},\tau\right)/\partial\tau > 0 \\ \partial B\left(\overline{\theta}\right)/\partial\bar{\mu} &= -\int_{\underline{\theta}}^{\overline{\theta}} (1+\Omega(\theta))\,\theta c(q^*)\,(\partial q^*/\partial\bar{\mu})\, f(\theta)\, d\theta \geq 0 \end{aligned}$$

- if the budget constraint is not binding, the result is immediate:

$$B\left(\overline{\theta}\right) \geq 0 \Rightarrow \bar{\mu} = 0 \Rightarrow \partial\bar{\mu}/\partial\tau = 0$$

Let us now prove that the gap between the ex post efficient abatement and the solution to (1), that is, $q^{**} - q^*$, is weakly decreasing with τ.

We can use Proposition 3 to say that $q^* = q^{**}$ for all if $\tau \geq \tilde{\tau}$. So, in this domain, the two solutions are the same. This is because the budget constraint is not binding.

On the contrary, when it is binding (when $\tau < \tilde{\tau}$), we can use the above results to obtain q^* as a function of θ and τ, that is, $q^*(\theta,\bar{\mu}(\tau))$ and to show that $\partial q^*(\theta,\bar{\mu}(\tau))/\partial\tau = (\partial q^*/\partial\bar{\mu})(\partial\bar{\mu}/\partial\tau) \geq 0$.

Now, if the fee is large enough, the ex post optimal allocation can be implemented ; it is thus solution to (1). An important result is that this is valid for all tax up to the critical value defined in proposition 3. This means that the choice of a tax in this region is only a distributional feature, as it does not make any difference from the point of view of efficiency.

Here, it is important to remember that the affectation principle has been precisely justified on a distributional field. When the pigouvian tax appears to be unacceptable to the firms, this mechanism is often presented as a good alternative: it favours the agents without efficiency cost. As far as these arguments are concerned, proposition 3 shows that this justification is valid. Indeed, it is possible to obtain the same result (the ex post optimal allocation) with a smaller tax.

7. INSTITUTIONAL PERSPECTIVE

In this section, we address the question of the institutional justification of the regulation of water pollution in France. The line of reasoning is inspired by Williamson's works [11,12].

The french government abandoned a piece of his fiscal power to water agencies. As far as the creation and the development of these institutions imply increased administrative costs, this choice must be justified by savings elsewhere. A reasonable opinion is that these benefits potentially lie in a closer proximity of the agencies to their constituents. It allows an easier communication and the use of more complex contractual schemes, that is, policies that could not be used by the government. In this respect, Proposition 4 gives a deeper description of the shape that could take the affectation principle in practice.

PROPOSITION 4. The optimal solution to (1) can be decentralized with:
1) if the budget constraint is binding:
a) a non linear subsidy of the abatement effort $t^*(q)$, increasing and convex,
or
b) a menu of linear subsidies $t(\theta, q) = \alpha(\theta) q + \beta(\theta)$, with $\alpha'(\theta) \leq 0$ and $\beta'(\theta) \geq 0$.
2) if the budget constraint is not binding, a unique linear subsidy $t(q) = (\tau^* - \tau) q + \beta$.

Proof. See the Appendix.

Using Proposition 4, we argue that, with respect to the argument proposed above, the creation of a specific agency to regulate water pollution using the affectation principle is only justified when the budget is binding. Indeed, in such a case, the regulation requires a complex process (precisely, non linear fiscal incentives). An explicit communication between the regulator and the firms is thus necessary. Firstly, the regulator must explain to the firms how to use the process. Secondly, the firm must inform the regulator about his participation in it. Such a scheme could not be used at a highly centralized level.

If the budget is not binding, the justification is much hardier. In such a case, no specific communication is necessary, since the regulation takes the form of a constant price. More precisely, it simply requires to subsidize abatement effort at a rate τ^*. So, if the creation of water agencies need to be justified, one must find other arguments (when the budget is not binding) than the idea of a simpler and more effective communication at a highly decentalized level.

The following table illustrates the allocation of remits in the french water regulation between the concerned institutions :

	Ministère	Préfet	DRIRE	Agences
Edict Norms	√	√	√	
Monitor			√	
Penalize		√		
Collect tax				√
Allocate aids				√

Legend: √ means "endowed with the right to"
DRIRE : Direction régionale de l'industrie, de la recherche et de l'environnement
Agences : Agence de l'eau et Agence de l'environnement et de la maîtrise de l'énergie (ADEME)

FIG.3. Power organization chart of french water regulation

APPENDIX

Proof of Proposition 2. Proof of part 1). Let us define:

$$\phi(\theta) = \tau^* - (1 + \bar{\mu}(1 + \Omega(\theta)))\theta c'(\rho(\theta, \tau))$$

An interior solution to (9) exists iff $\phi(\theta) \geq 0$ (this is a consequence of the second order condition). So, in the domain where $\phi(\theta) \geq 0$ (resp. $\phi(\theta) < 0$), the agent is (resp. is not) regulated and the optimal abatement is an interior solution to (9) (resp. equal to $\rho(\theta, \tau)$).

Remember that $\rho(\theta, \tau) = \arg\max_q (\tau q - \theta c(q))$. So we have $\theta c'(\rho(\theta, \tau)) = \tau$ and:

$$\phi(\theta) = \tau^* - (1 + \bar{\mu}(1 + \Omega(\theta)))\tau$$

If $\Omega(\theta)$ is non decreasing with θ, then $d\phi(\theta)/d\theta = -\bar{\mu}\tau d(\Omega(\theta))/d\theta \leq 0$.

The end of the proof is a direct consequence of this result:
- If $\phi(\underline{\theta}) \leq 0$, then $\phi(\theta) \leq 0$ for all θ. But $\phi(\underline{\theta}) \leq 0$ is equivalent to $\tau^*/\tau \leq 1 + \bar{\mu}$. In such case, we define $\tilde{\theta} = \underline{\theta}$.
- If $\phi(\overline{\theta}) \geq 0$, then $\phi(\theta) \geq 0$ for all θ. But $\phi(\overline{\theta}) \geq 0$ is equivalent to $\tau^*/\tau \geq 1 + \bar{\mu}\left(1/\left(\overline{\theta} f\left(\overline{\theta}\right)\right)\right)$. In such case, we define $\tilde{\theta} = \overline{\theta}$.
- If $\phi(\underline{\theta}) \geq 0$ and $\phi(\overline{\theta}) \leq 0$, then there exists $\tilde{\theta} \in \Theta$ solution to $\phi(\tilde{\theta}) = 0$. So $\tilde{\theta}$ solves $\Omega(\theta) = (\tau^*/\tau - 1)/\bar{\mu} - 1$.

The proof is complete when we note that, using the above definition of $\tilde{\theta}$, we have:

$$\theta \gtreqless \tilde{\theta} \Leftrightarrow \phi(\theta) \lesseqgtr 0, \text{ for all } \theta \in \Theta$$

Proof of part 2). The proof uses the implicit function theorem and the following facts:

- the second order condition:

$$\partial^2 H/\partial q^2 = -\left(1+\bar{\mu}\left(1+\Omega\left(\theta\right)\right)\right)\theta c''\left(q\right)f\left(\theta\right) \leq 0,$$

- the implementability constraint (for an interior solution, that is, use $\partial H/\partial q = 0$ and $\eta = 0$):

$$\partial^2 H/\partial q\partial\theta = -\left(1+\bar{\mu}\left(1+d\left(F\left(\theta\right)/f\left(\theta\right)\right)/d\theta\right)\right)c'\left(q\right)f\left(\theta\right) < 0 \text{ if } d\left[F\left(\theta\right)/f\left(\theta\right)\right]/d\theta \geq 0,$$

- the social productivity of abatement decreases with the social cost of the constraint (BB):

$$\partial^2 H/\partial q\partial\bar{\mu} = -\left(1+\Omega\left(\theta\right)\right)\theta c'\left(q\right)f\left(\theta\right) < 0.$$

For an interior solution, using the implicit function theorem, we get:

$$\begin{aligned}\partial q^*\left(\theta,\bar{\mu}\right)/\partial\theta &= -\partial^2 H/\partial q\partial\theta/\partial^2 H/\partial q^2 < 0\\ \partial q^*\left(\theta,\bar{\mu}\right)/\partial\bar{\mu} &= -\partial^2 H/\partial q\partial\bar{\mu}/\partial^2 H/\partial q^2 < 0\end{aligned}$$

For a corner solution, that is $q^*\left(\theta,\bar{\mu}\right) = \rho\left(\theta,\tau\right)$, one has:

$$\partial q^*\left(\theta,\bar{\mu}\right)/\partial\theta = -c'\left(\rho\left(\theta,\tau\right)\right)/\left(\theta c''\left(\rho\left(\theta,\tau\right)\right)\right) < 0$$
$$\partial q^*\left(\theta,\bar{\mu}\right)/\partial\bar{\mu} = 0$$

This completes the proof of the proposition.

Proof of Proposition 3. General idea: a first lemma shows that any contract that only mimics the statu quo is feasible (that is, it induces participation and truth-telling) and let the principal with a strictly positive budget (since no subsidy is necessary). A corollary is that, if the fee is equal to the marginal benefit of abatement, the principal can implement the ex post optimal abatement with a strictly credit budget. The end of the proof shows that, whatever the abatement that the principal implements, the budget diminishes when the fee is reduced. A critical value of the fee is obtained when the budget equals zero with the use of the ex post optimal abatement.

Remember that τ denotes the fee and $\rho\left(\theta,\tau\right)$ the associated statu quo abatement. Suppose that the principal implements $\rho\left(\theta,\tau\right)$. Since this abatement is (IC) in the statu quo, it is obviously (IC) in the contractual scheme even if no subsidy is distributed. This is what the next lemma says.

LEMMA A1. $C = \left\{(\rho(\hat{\theta},\tau),0) : \hat{\theta}\in\Theta\right\}$ is feasible and let $B\left(\overline{\theta}\right) > 0$.

Proof. The profit is given by $\pi = -\tau\left(z-q\right) - \theta c\left(q\right) + A$. A total differenciation yields:

$$\dot{\pi} = \left(\tau - \theta c'\left(q\right)\right)\dot{q} - c\left(q\right) + \dot{A}$$

But remember that (IC) implies that $\dot{\pi} = -c(q)$. So we conclude that:

$$0 = (\tau - \theta c'(q))\, \dot{q} + \dot{A}$$

Now, let us assume that the principal implements $\rho = \arg\max_q (\tau q - \theta c(q))$. Since $\tau = \theta c'(\rho)$, we get $\dot{A} = 0$ and we deduce that this abatement is implementable with a constant subsidy.

But note that $A = 0$ is sufficient to induce participation: it leaves all types with the same utility. This proves the first part of the proposition.

The second part of the proposition is obvious. We have $B(\overline{\theta}) = E_\theta[\tau(z-\rho) - A]$. Since $A = 0$ is sufficient to implement ρ and induce participation, this abatement lets a strictly credit budget.

Remember that $q^{**}(\theta)$ is the ex post optimal abatement and that the agent chooses it under the statu quo if $\tau = \tau^*$, that is, $\rho(\theta, \tau^*) = q^{**}(\theta)$.

COROLLARY A1. $C = \left\{(q^{**}(\theta), 0) : \hat{\theta} \in \Theta\right\}$ is solution to the principal's program when the fee τ is set at τ^*.

Proof. Let us test C as a solution to (1). The Lemma A1 proves that C is feasible and lets a strictly positive budget. So C is a candidate to solve it (all constraints are satisfied).

Furthermore, $q^{**}(\theta)$ satisfies (9) when $\bar{\mu} = 0$.

Since $q^{**}(\theta)$ is implemented with $B(\overline{\theta}) > 0$ when $\tau = \tau^*$, the corollary is proved.

So, q^{**} solves (1) when $\tau = \tau^*$. Now, let us assume that the principal still implements q^{**} but reduces the fee with respect to τ^*. This abatement remains the optimal solution to (1) as long as the budget remains positive. Thereafter, we find the critical value of the fee defined in proposition 3, below which the budget becomes negative.

LEMMA A2. (BB) can be written:

$$B(\overline{\theta}) = -\max_q \left[-\tau(z-q) - \overline{\theta} c(q)\right] - \int_{\underline{\theta}}^{\overline{\theta}} (1 + \Omega(\theta))\, \theta c(q)\, f(\theta)\, d\theta$$

Proof. We begin with $B(\overline{\theta}) = E_\theta[\tau(z-q) - A] = E_\theta[-\pi - \theta c(q)]$ since we have: $\pi = -\tau(z-q) - \theta c(q) + A$. Using (IC), which implies that $\pi(\theta) = \pi(\overline{\theta}) + \int_\theta^{\overline{\theta}} c(q)\, ds$, and (IR), which implies that $\pi(\overline{\theta}) = R(\overline{\theta}, \tau)$, we get:
$B(\overline{\theta}) = -R(\overline{\theta}, \tau) - E_\theta\left[\int_\theta^{\overline{\theta}} c(q)\, ds + \theta c(q)\right]$. An integration by parts yields:

$$E_\theta\left[\int_\theta^{\overline{\theta}} c(q)\, ds\right] = E_\theta[c(q)\, F(\theta)/f(\theta)]$$

So, we finally obtain $B(\overline{\theta}) = -R(\overline{\theta}, \tau) - E_\theta[(1 + \Omega(\theta))\, \theta c(q)]$.

The proof is now quite complete. Suppose that the principal implements q^{**}. Introduce it into the expression of the budget. If $\tau = 0$, then $B\left(\overline{\theta}\right) < 0$. If $\tau = \tau^*$, lemma A1 shows that $B\left(\overline{\theta}\right) > 0$. Using the enveloppe theorem on $B\left(\overline{\theta}\right)$ defined in lemma A2, on gets $\partial B\left(\overline{\theta}\right)/\partial\tau = z - \rho > 0$. So there exists $\tilde{\tau} \in]0, \tau^*[$ such that $B\left(\overline{\theta}\right) = 0$. Whatever $\tau \geq \tilde{\tau}$ (resp. $\tau < \tilde{\tau}$), $B\left(\overline{\theta}\right) \geq 0$ (resp. $B\left(\overline{\theta}\right) < 0$), so q^{**} is (resp. is not) solution to (1).

Proof of Proposition 4.

We know that (IC) and (IR) imply, for any decreasing abatement $q(\theta)$:

$$A(\theta) = R\left(\overline{\theta}, \tau\right) + \int_{\theta}^{\overline{\theta}} c(q)\, ds + \tau(z - q) + \theta c(q)$$

Proposition 2 proves that the optimal abatement q^* is a strictly decreasing function of the type θ. This function can thus be inverted to obtain a function $\theta = \theta^*(q)$, that gives the agent's type as a function of the abatement level that he chooses. The optimal subsidy is thus:

$$t^*(q) = R\left(\overline{\theta}, \tau\right) + \int_{\theta^*(q)}^{\overline{\theta}} c(q)\, ds + \tau(z - q) + \theta^*(q)\, c(q)$$

This subsidy is said to be optimal because it induces the agent (when he faces it) to choose the optimal abatement effort, solution to (1).

We now prove part 1) a) of the proposition, that is, that the optimal subsidy is increasing and convex.

A total differenciation of $t^*(q)$ with respect to q yields:

$$t^{*\prime}(q) = \theta^*(q)\, c'(q) - \tau = \max\left(\tau^*/\left(1 + \bar{\mu}\left(1 + \Omega\left(\theta^*(q)\right)\right)\right) - \tau, 0\right) \geq 0$$

since (9) implies that $\theta^*(q)\, c'(q) = \max\left(\tau^*/\left(1 + \bar{\mu}\left(1 + \Omega\left(\theta^*(q)\right)\right)\right), \tau\right)$. Finally, a second differenciation leads to:

$$t^{*\prime\prime}(q) \geq 0$$

because $t^{*\prime\prime}(q)$ is either equal to 0 (if $\bar{\mu} = 0$) or to:

$$-\bar{\mu}\tau^*\Omega'\left(\theta^*(q)\right)\theta^{*\prime}(q)/\left(1 + \bar{\mu}\left(1 + \Omega\left(\theta^*(q)\right)\right)\right)^2 \geq 0$$

if $\bar{\mu} > 0$ given that $\Omega'(\theta) \geq 0$).

We now prove part 1) b) of the proposition.

Because $t^*(q)$ is convex, it can be replaced by the family of its tangents (see Laffont and Tirole [5], proposition 1.4, page 69). It is thus possible to define a menu of linear contracts $t(\theta, q) = \alpha(\theta) q + \beta(\theta)$, with $\alpha(\theta) = t^{*\prime}(q^*(\theta))$ and $\beta(\theta) = t^*(q^*(\theta)) - t^{*\prime}(q^*(\theta)) q^*(\theta)$. The properties of $\alpha(\theta)$ and $\beta(\theta)$, given by:

$$\begin{aligned} \alpha'(\theta) &= t^{*\prime\prime}(q^*(\theta)) \partial q^*(\theta) / \partial\theta \leq 0 \\ \beta'(\theta) &= -t^{*\prime\prime}(q^*(\theta)) q^*(\theta) \partial q^*(\theta) / \partial\theta \geq 0 \end{aligned}$$

follow directly from the convexity of the envelope $t^*(q)$ and the property of the optimal solution.

Part 2) of the proposition is a particular case of the above results, when the budget constraint is not binding (i.e. $\bar{\mu} = 0$). If $\bar{\mu} = 0$, α and β are independant of θ (see above). More precisely, one can check that $\alpha = t^{*\prime}(q) = \tau^* - \tau$.

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